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SPEED OF MOVEMENT OF THE INSTANTANEOUS CENTER OF ROTATION ON THE FIXED CENTROID

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Abstract: *In the paper, the components of the speed of movement of the instantaneous rotation center and its size relative to the fixed reference system were established. As an example, the aforementioned expressions for the antiparallel mechanism were determined.*
Keywords *kinematics of the plane-parallel motion of the rigid, centroid, relative centroid.*

1.INTRODUCTION

In the kinematics of the planar-parallel motion of the rigid, in the plane of motion, at one point, the position of a point that has zero speed relative to the reference system in solidarity with the rigid one can be determined. This point, called the instantaneous center of rotation (ICR), generates with respect to the fixed reference system $(x_1 O_1 y_1)$ a curve called **fixed centroid** (FC) or base, and to the reference system in solidarity with the rigid (xOy) another curve. called the **centroid mobile** (MC) or rolling (see figure 1).

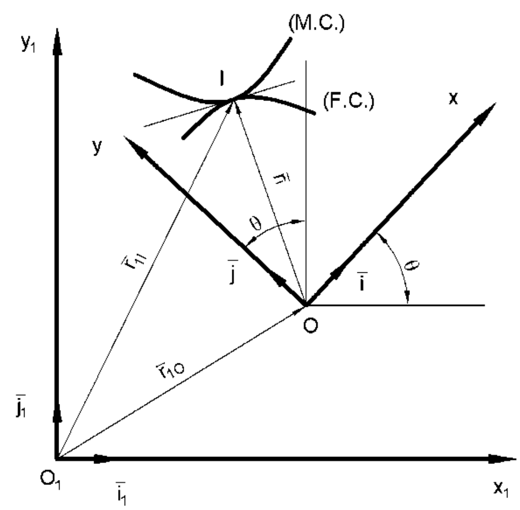

Figure 1: The reference systems in the plane-parallel motion

The movable centroid rolls without sliding on the fixed centroid. If at the center of the instantaneous rotation we have a perpendicular to the plane of motion, we obtain the instantaneous axis of rotation, which generates with respect to the two reference systems two cylindrical surfaces called: fixed axoid with respect to the fixed reference system, respectively movable axoid with respect to the reference system in solidarity with the rigid. Similarly, the movable axoid rolls without sliding on the fixed axoid.The velocity distribution in the plane-parallel motion is similar at one point, with the velocity distribution from the rigid in the rotational motion around a fixed axis, with the difference that the instantaneous axis of rotation changes its position at the next moment. This property simplifies the solution of the kinematic problem.

2. INSTANT ROTATION CENTER (IRC)

The coordinates of the instantaneous center of rotation with respect to the system of solidarity with the rigid (xOy) have the expressions:

$$\xi = -\frac{v_{Oy}}{\omega} \tag{1}$$

$$\eta = \frac{v_{Ox}}{\omega} \tag{2}$$

In which v_{Ox} and v_{Oy} are the components of the velocity of the mobile origin. [1-3].Compared to the fixed reference system, the coordinates of the instantaneous center of rotation have the expressions [1,2]:

$$\xi_1 = x_{10} - \frac{v_{Oy_1}}{\omega} = x_{10} - \frac{\dot{y}_{10}}{\omega} \tag{3}$$

$$\eta_1 = y_{10} + \frac{v_{Ox_1}}{\omega} = y_{10} + \frac{\dot{x}_{10}}{\omega} \tag{4}$$

Similarly x_{10}, y_{10}, $\dot{x}_{10}$, $\dot{y}_{10}$, are the coordinates, respectively, of the velocity of origin O of the reference system xOy with respect to the fixed reference system. We propose in the following, to establish the components of the IRC displacement speed on the fixed centroid and to detail them in the case of the antiparallelogram mechanism, determining in this case the magnitude of the above mentioned speed.

3. THE SPEED OF THE INSTANT ROTATION CENTER ON THE FIXED CENTROID

By deriving with respect to the time of relations (3) and (4), the components of the speed of movement of the center of the instant of rotation center on the fixed centroid are obtained:

$$v_{Ix_1} = \dot{\xi}_1 = \dot{x}_{10} - \frac{\ddot{y}_{10}\,\omega - \dot{y}_{10}\,\varepsilon}{\omega^2} \tag{5}$$

$$v_{Iy_1} = \dot{\eta}_1 = \dot{y}_{10} + \frac{\ddot{x}_{10}\omega - \dot{x}_{10}\varepsilon}{\omega^2} \tag{6}$$

respectively the size of this speed:

$$v_{1I} = \sqrt{v_{Ix_1}^2 + v_{Iy_1}^2} = \sqrt{\dot{\xi}_1^2 + \dot{\eta}_1^2} \tag{7}$$

4. EXAMPLE OF CALCULATION

Next we will determine the components and the magnitude of the IRC displacement speed on the fixed centroid, within the antiparallelogram mechanism. In the collections of mechanics problems [4-6], among the proposed problems, we find the case of the determination of centroid for the antiparallelogram mechanism having a small fixed side.

In figure 2 we present the antiparallelogram mechanism at which the motor element is the crank O_1A, which rotates evenly, with the angular speed ω_0 ($\theta = \omega_0 t$). We have noted $O_1O_2 = AB = 2b$ and $O_1A = O_2B$ =2l, ($2l > 2b$).In this case the fixed and mobile centroid are two congruent ellipses with foci at the points O_1 *și* O_2 and respectively A and B and the center of the instantaneous rotation is at the intersection of the bars O_1A *and* O_2B We chose the points O_1 and A and It has the origins of the reference system: fixed ($O_1O_2 = O_1x_1$) and that of solidarity with the bar AB (AB = Ox)

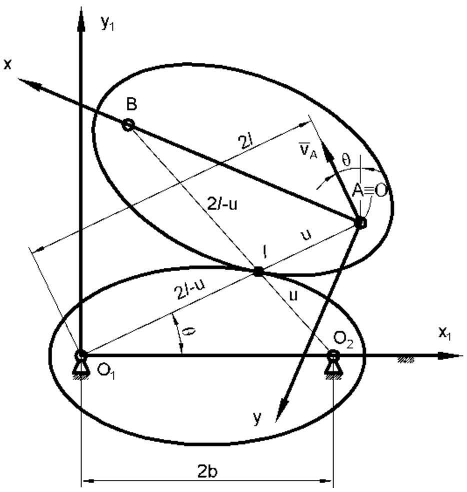

Figure 2: Antiparallelogram mechanism - centroidele

$$x_{10} = O_1A\cos\theta = 2l\cos\theta \tag{8a}$$

$$y_{10} = O_1A\sin\theta = 2l\sin\theta \tag{8b}$$

$$\dot{x}_{10} = -2l\,\omega_o\,sin\theta \tag{9a}$$

$$\dot{y}_{10} = 2l\,\omega_O\,cos\theta \tag{9b}$$

$$\ddot{x}_{10} = -2l\,\omega_O^2\,cos\theta \tag{10a}$$

$$\ddot{y}_{10} = -2l\,\omega_O^2\,sin\theta \tag{10b}$$

From the relationship: $v_A = 2l\,\omega_O = IA\,\omega = u\,\omega$ the angular velocity ω of the bar is set AB:

$$\omega = \frac{2l\,\omega_O}{u} \tag{11}$$

In the isosceles trapezoid $O_1B\,A\,O_2$we have the congruent triangles I O_1O_2 respectively IBA. Applying the generalized Pythagorean theorem in the triangle I O_1O_2 and taking into account the relation $O_1I = (2l - u)$ we obtain:

$$u = \frac{l^2 + b^2 - 2lb\cos\theta}{l - b\cos\theta} \tag{12}$$

respectively

$$\omega = \frac{2l\omega_O(2l - b\cos\theta)}{l^2 + b^2 - 2lb\,cos\theta} \tag{13}$$

By derivation the relation (13) with respect to time it results:

$$\varepsilon = \dot{\omega} = -\frac{2lb(l^2 - b^2)\omega_O^2\,sin\theta}{(l^2 + b^2 - 2\,lb\,cos\theta)^2} \tag{14}$$

If relations (9a, b), (10 a, b), (13) and (14) are introduced in (5) and (6) by elementary calculations, the components of the search speed are obtained:

$$v_{Ix_1} = -\frac{l(l^2 - b^2)\omega_O}{(l - b\,cos\theta)^2} \tag{15}$$

$$v_{Iy_1} = \frac{(-b + l\,cos\theta)\omega_o}{(l - b\,cos\theta)^2}(l^2 - b^2) \tag{16}$$

and by introducing these relations in (7) we obtain the magnitude of the instantaneous rotational center displacement speed on the fixed centroid:

$$v_{1I} = \frac{(l^2 - b^2)\omega_O}{(l - b\,cos\theta)^2}\sqrt{l^2 + b^2 - 2lb\,cos\theta} \tag{17}$$

We can consider the use of gears with non-circular wheels as an application in industry and technique of the aspects related to centroid, from the kinematics of the plane-parallel movement of the rigid. In the specialized literature [7,8] the conjugated profiles of the non-circular (possibly ellipsoidal) wheels are called **centroid.** Since both wheels are in motion, we believe that the conjugated profiles should be referred to as **relative centroid,** and the point of contact between the wheels as the instantaneous center of relative rotation, (ICRR) so if we consider that one of the wheels is fixed, then the speed of the point of contact between the two wheels is even the speed of movement of the instantaneous center of rotation on the fixed centroid.

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