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THE CONTACT PROBLEM BETWEEN ELASTIC SOLIDS WIYH SMAL STRAINS AND DEFORMATIONS

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Abstract: *The boundary element formulation and the computer implementation of the 2-D contacvt problem with small displacements and strains between elastic anisotropic materials are presented in this paper.The contact program include isoparametric linear, quadratic,and quater-point-traction-singular elements. several contact zones with different friction coefficients berween the solids. Several exemples have been included, specially the computation od contact tractions in composite material plates with bolted joints or the influence on the stress intensity factor of the crack closure effects.*
Keywords:*contact, tension,composite,boundary,element*

1. INTRODUCTION

Over the last few years important advancements have been made in the inclusion of contact formulations into standard finite element, or boundary element programs. This last method seems to have proved advantageous in treating the linear contact problem, taht is the contact between lineart elastic solids with small displacements and strains, as occurs for instance along the crack lips of elastic bodies.
The formulation of the BEM is primarily incuded for completeness, so are the formulation and algoritma used to solve the contact problems between two solids. Finally several exemples are explained in detail, specially the study of contact traction in bolted joints in composite laminates.

2. FORMULATION OF THE CONTACT PROBLEM BETWEEN ELASTIC PROBLEM WITH SMALL STRAINS AND DISPLACEMENTS

The unilateral contact problem with small displacements and strains is just a linear elastic problem for each solid under non-linear and initialy unknown boundary conditions, along an unknown contact surface. These conditions depend on load level and the deometry of solids in contact.
In this case, only the contact problem between two elastic solids will be considered. The extensionto multibody problems or the particularization to rigid base problems are straightforward once the above formulation has been obtained, regardless of the implementation and modelling difficulties impled. Let $\delta\Omega$ be the boundary neighbourhood of a point P on the contact surface, and $n=f_A(t)$, $n=f_B(t)$ the equations that represent the undeformed surfaces for both solids A, B along the contact zone, in a local coordinates system tangent-notmal to the surface $\delta\Omega_A^c$, $\delta\Omega_B^c$,(if a small displacement problem is considered those equations must be essentially the same, and also similar to the equation of the final contact zonbe after the deformation $\delta\Omega^c$) (Fig.1)
The non-penetration condition at point P is established as

$$d_N + u_N \leq 0 \quad \textbf{(1)}$$

with d_N the projection of the initial vector joining the equivalent points P, P' (same position after the contact) along the normal to $\delta\Omega^c$, and u_N the projection of the relative displacement beween the two points along the same normal.

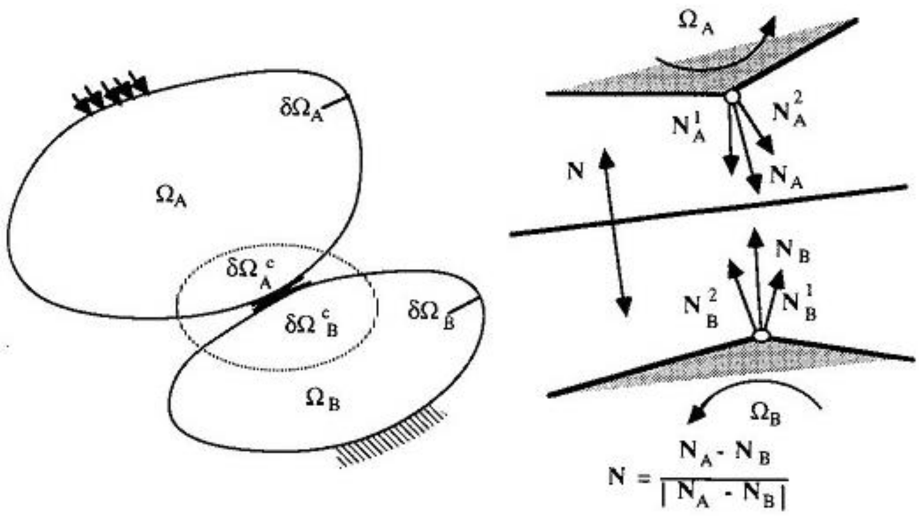

Figure 1 Intermediate normal between the two solids

It can shown that conditions (1) is no more than the linerized non-penetration condition for large displacements along the coordinate t in the neighbourhood of the point P. This kinematic expression is then the approximated real condition and it is exact only when both solids have the same normal at the points P and P' which remain fixed along the contact process, and the relative displacement has the direction of that normal. When the normals to both solids are orthogonal, the error is maximum, even marking is possible to violate the non-penetration condition. However, cases close to this are unreal in practical in practical small displacement contact problems.
The static boundary conditions, in the unilateral case, wich a Coulomb friction law, as the one considered in this paper, can be expressed as

$\sigma_N \leq 0 \qquad \tau \leq \mu \cdot \sigma_N$ with μ the friction coefficient **(2)**

The direction N that has been used to project the displacements and tractions in order to impose the boundary conditions , is the average between the two normals to both solids in correspondiong boundary nodes (Fig.1)
Besides those kinematic and static conditions for each domain, the equilibrium and compatibility conditions between both solids in both solids must be fulfilled along the contact zone. In order to do this, different zones along the boundaries are defined (Fig.2):

- Out of contact zone (zone 1). It is the one that is never in contact;
- Candidate to contact zone (zone 2). It is the one which is not in contact yet, but can be cantact for a certain load level;
- Sliding zone (zone 3), $|\tau| = \mu \cdot \sigma_N$;

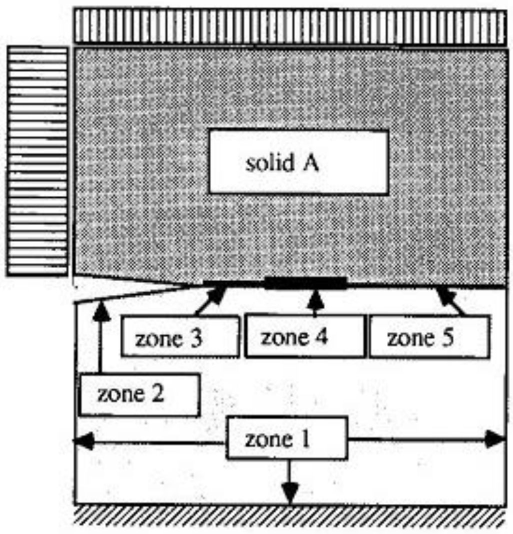

Fig2.Contact zones

- Adhesive zone (zone 4), $|\tau| \prec \mu \cdot \sigma_N$;
- Welded zone (zone 5). It is really a contact zone, but an interface between two domains, but has been included as a contact zone in order to generalize the program including the possibility of bilateral contact problems with zones under tension.

The contact problembetweem two solids consists then in establishment of the BE equations for each of the solids under contact, including implicity or explicity thr boundary conditions (equilibrium and compatibility) along the contact region for each load level, and the standard boundary regions.

3. TYPES OF ELEMENTS AND NODES

The studie that implements the above formulation includes linear and quadratic isoparametric and singular traction quater-point continuous elements, with coner nodes in zone 1 being treated as in Alarcon et all[1]. For friction problem a different friction coefficient can be defined for each element so that different friction properties.

The shape functions for linear and quadratic isoparametric elements are well-known, while for singular storage is not needed since all the integration constant are included in the complete matrix and their positions also remain unchanged. This method can be implemeted very easily in a standard BE code.

Here this second option has been chosen, while, use the first one. The reasons shown above can be supplemenred by the fact that an adequate positioning of the unknowns and equations can be used in order to reduce the number of them that have to be triangularized at each step.

As a conclusion, it can be said that both approaches are almost equivalent with respect to the CPUand the memory needed, but the one used here is simpler with respect to the assemblage and more difficult with regards the solving process.

The first step to be taken, after having selected the method, is a static condensation process, in order to eliminated the unknowns corresponding to the nodes out of the candidate to contact zone (node 11). If a standard multidomain BE method is used for both bodies, this static process can be writen as:

$$K_{LL}^S \cdot x_L^S + K_{LC}^S \cdot x_C^S = V_L^S \qquad \textbf{(3)}$$
$$K_{CL}^S \cdot x_L^S + K_{CC}^S \cdot x_C^S = V_C^S$$

with S=A, B; x_L unknowns to eliminated, and x_C the corresponding to the candidate to contact zone.

Equations (3) can also be expressed as

$$\overline{K}^S \cdot x_C^S = \overline{V}^S \qquad \textbf{(4)}$$

Each of matrices K^S is a 2n x 6n matrix. with n number of nodes in the C zone (2 integral equations for each nodes as a collocation point for each solid, and 6 unknowns for each node;2 displacements, u_1, u_2; and 2 traction for the two elements to which it belongs, σ_{ant}, τ_{ant}, σ_{pos} and τ_{pos}).

Finnaly, 8 additional equations per equations per contacting node have to be included, which correspond to the contact matrix K^{AB} and which depend on the type of node. For instance for node type of node type 44 with continuous nornal, they are

$$u_1^A = u_1^B \qquad u_2^A = u_2^B \qquad \sigma_{ant}^A = \sigma_{pos}^A$$
$$\tau_{amt}^A = \tau_{pos}^A \; \sigma_{pos}^B = \sigma_{ant}^B \quad \tau_{pos}^B = \tau_{ant}^B \qquad \textbf{(5)}$$
$$\tau_{ant}^A = \tau_{pos}^B \qquad \tau_{ant}^A = \tau_{pos}^B$$

The structure adopted for matrix and vector of unknowns is shown in (Fig.3) the matrices K^A, K^B, and K^{AB} being the only ones stored.

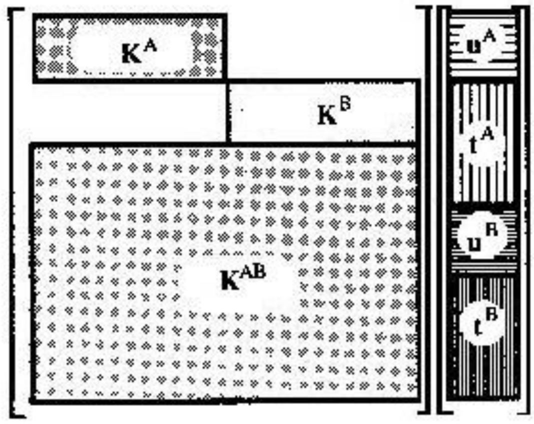

Fig.3.Structures of the system of equations

With respect to the solution method, the one used here is a standard Gaussian elimination scheme with row pivotting and with a pretriuangularization of the matrices K^A and K^B, or K^{AB}, which remain unchanged throughout the proces.With this, only a system of 5nx6n equations,(4nx4n in the first approach described before if the continuity of tractions is assumed), has to be triangularized for each increnent step.

4. CONCLUSIONS

It has been shown that the B.E.M. may be used to study the problem of propagating cracks in orthotropic bodies in a similar form to the previous works on isotropic materials. Also,the singular boundary elements firstly proposed by Blandford et.all. give very good results in the computation of stress intensity factors every coarse meshes, specially using a direct traction approach like the one presented by Martinez and Dominguez, being only necessary the modification of the fundamental solution of standard isotropic boundary element program.

REFERENCES

[1] F. Erdogan, Crack Propagation Theory. Facture II, (Edited by H. Liebowitz), pp. 498-592. Academic Press. Ne York (1969).
[2] C.T. Herakovich, Fracture of fibrous Composites AMD Vol. 74 ASME (1986).
[3] R.N.L.Smith, Developments in the BEM for the Solution of Elastic Fracture Problems.Proc.B.E.M. X Int Conf.(edited by C.A.Brebbia) pp.155-176. Spinger, Berlin (1988).
[4] A. Piva and E. Viola, Crack propagationin an orthotropic medium.Engng Fracture Mech.29, 535-548 (1988).
[5] C.A.Brebbia, The boundary Element Method for Engineers,Pentch Press, Computational Mechanics Publication, Boston (1978).
[6] M. Doblare, Computational Aspects of the B.E.M. Topics in B.E. ResearchesIII, (edited C.A.Brebbia), pp.51-131,Springer, Berlin (1987).