Transilvania University of Brasov
FACULTY OF MECHANICAL ENGINEERING

COMEC 2021

Brașov, ROMANIA, 21-23 October 2021

THE CHANGE OF THE NATURAL FREQUENCIES FOR A CONTINUOUS BEAM WITH THREE SPANS IN THE PRESENCE OF TRANSVERSAL CRACKS

Z.I. Praisach[1], G.R. Gillich[1], D. Pîrșan[1], D. Ardeljan[1], V. Paşcu[1]
[1] Babeş-Bolyai University, Faculty of Engineering, Reşiţa, România, zeno.praisach@ubbcluj.ro, gilbert.gillich@ubbcluj.ro, dan.pirsan@ubbcluj.ro, dorel.ardeljan@ubbcluj.ro, viorel.pascu@ubbcluj.ro

Abstract: *The paper presents the analytical relationship for the changes of the natural frequencies at the occurrence of a transversal damage on a continuous beam with three spans using the Euler-Bernoulli theory, regardless of the location of the damage on the structure. When the damage appears on the beam, the changes of the natural frequencies can be highlighted by the square of the mode shape curvature for any location of the damage on the structure. The paper exemplifies the changes in natural frequencies for two possible locations of the damage.*
Keywords: *natural frequency, strain energy, mode shape*

1. INTRODUCTION

Vibration-based damage detection bases on the fundamental idea that when damage appears in a structure, changes appear in the geometry of the structure [1] and in their physical properties: stiffness, damping and mass. All these changes will be reflected in modal properties [2]: frequencies, mode shapes, modal curvatures and modal damping and can be easily detected. It was found that the methods based on the analysis of mode shape proved to be quite insensitive to incipient damage and by use of mode shape curvatures, these present an enhanced sensitivity to damage detection [3-5]. Advantages to use the modal shape curvature which represents the second derivatives of the mode shapes as features for damage detection methods are obvious. A first advantage lies in the content of the local information carried by the mode shapes and especially by their derivatives [6]. As a second advantage, lies in the low sensitivity to environmental effects of the mode shape than the natural frequencies. Discontinuities in form of transversal cracks affect the beam stiffness and consequently its dynamic behavior, by reducing the values of natural frequencies and changing the mode shapes and curvatures [7-8]. For effective damage detection it is important to assess as accurate as possible the small frequency changes [9]. The paper presents in analytical form the relationship that gives us the change of natural frequencies to the appearance of a transversal crack for a continuous beam with three spans, regardless of the position where the defect appears, by using the modal shapes curvature method. It extends an approach [10].of calculating the modal parameters for two-span beams.

2. ANALYTICAL APPROACH

From the literature it is known the characteristic function $W_i(x_i)$ or normal mode of span can be expressed generally:

$$W_i(x_i) = A_i \sin(a_n x_i) + B_i \cos(a_n x_i) + C_i \sinh(a_n x_i) + D_i \cosh(a_n x_i) \tag{1}$$

were,
i = 1, 2, 3 represents the number of spans;
A_i, B_i, C_i, D_i are the integration coefficients;
a_n is the eigenvalues;
$n = 1, 2, \ldots, \infty$ is the number of vibration mode.
By applying the boundary conditions (2) for the characteristic points 1, 2, 3 and 4, on a clamped - hinged continuous beam with three spans (fig. 1) and after solving the system obtained from the boundary conditions [9-11], results the normalized modal functions (3) that describe the mode shape of the structure.

$$\langle 1\rangle\begin{cases}W_1(0)=0\\W'_1(0)=0\end{cases};\quad \langle 2\rangle\begin{cases}W_1(l_1)=0\\W_2(0)=0\\W_1'(l_1)=W_2'(0)\\W_1''(l_1)=W_2''(0)\end{cases};\quad \langle 3\rangle\begin{cases}W_2(l_2)=0\\W_3(l_3)=0\\W_2'(l_2)=-W_3'(l_3)\\W_2''(l_2)=W_3''(l_3)\end{cases};\quad \langle 4\rangle\begin{cases}W_3(0)=0\\W_3''(0)=0\end{cases} \tag{2}$$

$$\begin{cases}W_1(x_1)=A_1\left[\sin(a_nx_1)-\sinh(a_nx_1)-\dfrac{\sin(a_nl_1)-\sinh(a_nl_1)}{\cos(a_nl_1)-\cosh(a_nl_1)}\left[\cos(a_nx_1)-\cosh(a_nx_1)\right]\right]\\W_2(x_2)=A_2\sin(a_nx_2)+B_2\left[\cos(a_nx_2)-\cosh(a_nx_2)\right]+C_2\sinh(a_nx_2)\\W_3(x_3)=A_3\left[\sin(a_nx_3)-\dfrac{\sin(a_nl_3)}{\sin(a_nl_3)}\sinh(a_nx_3)\right]\end{cases} \tag{3}$$

with $x_1\in[0,\ l_1],\ x_2\in[0,\ l_2],\ x_3\in[0,\ l_3]$.

For an undamaged continuous beam, of constant section, the natural frequency (4) can be expressed:

$$f_n=\frac{a_n^{\ 2}}{2\pi}\sqrt{\frac{E\cdot I}{m\cdot L^4}} \tag{4}$$

where,

f_n [Hz] is the natural frequency;

a_n is the eigenvalues;

E [N/m^2] – Young’s modulus;

I [m^4] - moment of inertia;

m [kg] – beam mass;

$L = l_1 + l_2 + l_3$ [m] – beam length;

It is known that the deformation energy:

$$U_n=\frac{1}{2EI}\int_0^l W_n''^2\,dx \tag{5}$$

is directly proportional with the square of mode shape curvature and from the literature [5, 6, 7], the relation which permit the evaluation of the natural frequencies for a damaged beam (6) takes into acount also the square of mode shape curvature whose form is:

$$f_{Dn}=f_{Un}\left(1-\gamma\cdot W_n''^2\right) \tag{6}$$

where.

f_{Dn} [Hz] is the natural frequency for the damaged beam;

f_{Un} [Hz] is the natural frequency for the healthy beam;

γ is a coefficient for damage severity;

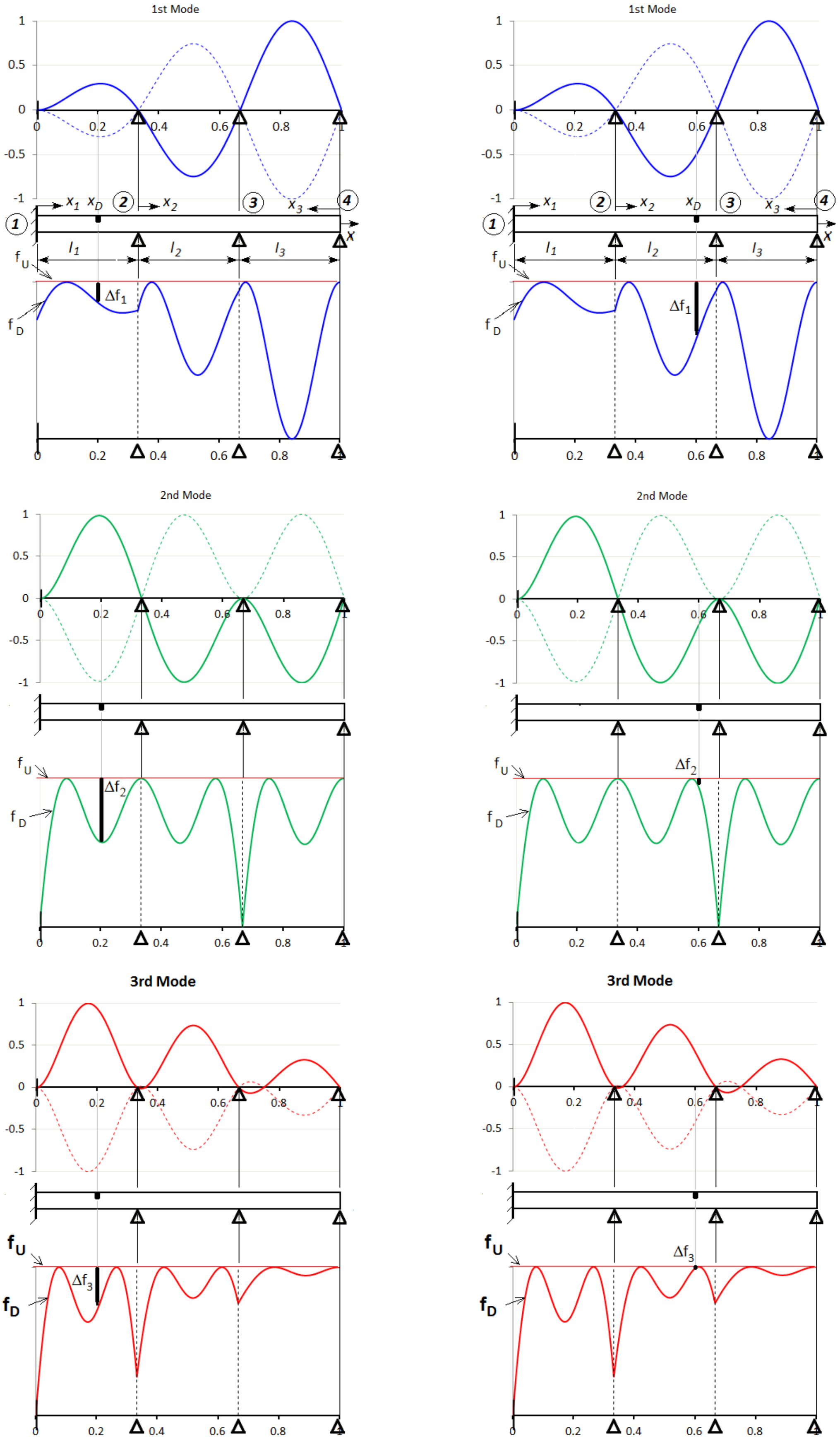
W_n'' is the normalized mode shape curvature, expre4ssed with relation (7).

$$\begin{cases}W_1''(x_1)=-a_n^{\ 2}\cdot A_1\left[\sin(a_nx_1)+\sinh(a_nx_1)-\dfrac{\sin(a_nl_1)-\sinh(a_nl_1)}{\cos(a_nl_1)-\cosh(a_nl_1)}\left[\cos(a_nx_1)+\cosh(a_nx_1)\right]\right]\\W_2''(x_2)=-a_n^{\ 2}\cdot\left[A_2\sin(a_nx_2)-B_2\left[\cos(a_nx_2)+\cosh(a_nx_2)\right]+C_2\sinh(a_nx_2)\right]\\W_3''(x_3)=-a_n^{\ 2}\cdot A_3\left[\sin(a_nx_3)+\dfrac{\sin(a_nl_3)}{\sin(a_nl_3)}\sinh(a_nx_3)\right]\end{cases} \tag{7}$$

It can be observed (fig. 1) that in the points where the deformation energy is zero, respectively in the inflection points of the modal functions (3), the second derivative of these functions are zero and the natural frequencies for the damaged beam (6) are equal to the natural frequency for the healthy beam [8, 9].

Taking into consideration the severity coefficient equal to the unit, the change of the natural frequencies for the three span continuous beam with a damage located anywhere on the entire beam, can be represented graphically (fig. 1) by the relation below:

$$\Delta f_n=f_{Un}\left(1-W''^2\right) \tag{8}$$


1st Mode
1st Mode
2nd Mode
2nd Mode
3rd Mode
3rd Mode
x1
xD
x2
x3
l1
l2
l3
x
fU
fD
Δf1
Δf2
Δf3

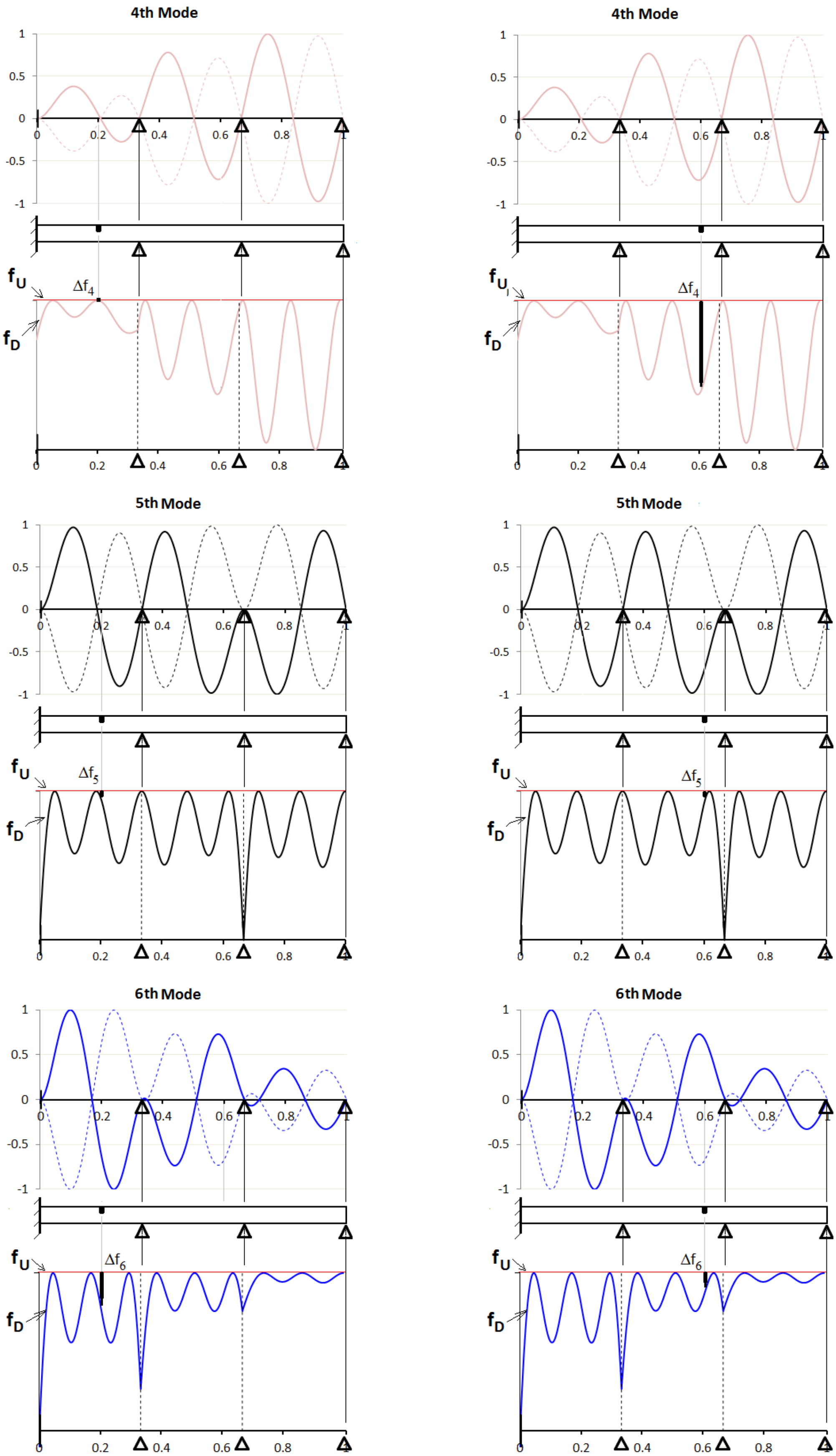


Figure 1: The first six mode shape for the transverse vibrations (top) and natural frequencies evolution (bottom) for a three-span continuous beam with clamped - hinged ends which has a crack located at *x/l*=0.2 from the left end (left diagrams) and with a crack located at *x/l*=0.6 from the left end (right diagrams).

Figure 1 shows the normalized modal functions for the first six vibration modes for a continuous beam with three span, where the intermediate joints are placed at equal distances, respectively $l_1 = l_2 = l_3$. Note that always $l_1 + l_2 + l_3 = 1$. The beam sketch is illustrated under the modal functions, and the change of natural frequencies in case of a transversal damage or crack appears on the continuous beam (for any location of the damage) can be followed under the beam sketch.
In the left column of figure 1 are highlighted the natural frequency changes for a transverse damage located at x/l=0.2 from the left end of the beam, and in the right column can be followed the natural frequency changes for a damage at x/l=0.6 from the left end of the beam.

3. CONCLUSION

Analyzing the changes of the natural frequencies for the first six vibration mode presented in figure 1, it can be concluded that the deformation energy or square mode shape curvature expressed by relation (7), along the entire continuous beam is different for each vibration mode. Thus, reducing the rigidity due to damage, the frequencies will decrease in different ways, depending on the damage location. This remark allows us to develop patterns able to characterize the effect of damage, to any location along the beam.
Patterns that characterize the damage location are consequently derived by using the squared mode shape curvatures of the healthy beam. In this case, the location of the damage becomes an inverse problem [7] and the location of the damage on the beam can be found by interpreting the results of the frequency measurements for the healthy and damaged condition.

REFERENCES

[1] Wei F., Qiao P., Vibration-based Damage Identification Methods: A Review and Comparative Study, Structural Health Monitoring 10(1), 2011, pp. 83-111.
[2] Doebling, S.W., Farrar, C.R., Prime, M.B., A summary review of vibration-based damage identification methods, Shock and Vibration Digest 30(2), 1998, pp. 91-105.
[3] Vlase S., Marin M., Öchsner A., Scutaru M.L., Motion equation for a flexible one-dimensional element used in the dynamical analysis of a multibody system, Continuum Mechanics and Thermodynamics 31(3), pp. 715-724, 2019.
[4] Ntakpe J.L., Praisach Z.I, Mituletu C.I., Gillich G.R., Muntean F., Natural frequency changes of two-span beams due to transverse cracks, Journal of Vibration Engineering & Technologies5(3), pp. 229-238, 2017.
[5] Khiem N.T., Khang P.X., Hang P.T., A method for calculating natural frequencies of multispan beam with arbitrary number of cracks, National conference on Engineering Mechanics Celebrating the 35th Anniversary of the Institute of Mechanics, VAST, Hanoi, 2014.
[6] Han L.B., Nguyen H.H., Xiang Y.M., Vibration analysis of a multi-span continuous beam with crack, Applied Mechanics and Materials 256-259, pp. 964-972, 2013.
[7] Gillich, G.R, Praisach, Z.I., Detection and Quantitative Assessment of Damages in Beam Structures Using Frequency and Stiffness Changes, Key Engineering Materials 569, pp. 1013-1020, 2013.
[8] Gillich G.R., Abdel Wahab M., Praisach Z.I., Ntakpe J.L., The influence of transversal crack geometry on the frequency changes of beams, International Conference on Noise and Vibration Engineering ISMA2014, paper ID 666, 2014.
[9] Ntakpe J.L., Gillich G.R., Mituletu I.C., Praisach Z.I., Gillich N. An Accurate Frequency Estimation Algorithm with Application in Modal Analysis, Romanian Journal of Acoustics and Vibration 13(2), pp. 98-103, 2016.
[10] Gillich G.R., Praisach Z.I., Iancu V., Furdui H., Negru I., Natural Frequency Changes due to Severe Corrosion in Metallic Structures, Strojniški vestnik - Journal of Mechanical Engineering 61(12), pp. 721-730, 2015.
[11] Gillich G.R., Ntakpe J.L., Wahab M.A., Praisach Z.I., Mimis M.C., Damage detection in multi-span beams based on the analysis of frequency changes Journal of Physics: Conference Series 842 (1), 012033, 2017.