Analysis of the eyeball model through the prism of the laws of Newtonian mechanics

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Abstract. *Applications of human visual function modeling are particularly useful in medical or facial recognition, communications or entertainment studies, but in any of the fields the modeling results shorten and optimize specific procedures, bringing an increase in efficiency. Optical, mechanical or electrical modeling's of the visual system, as well as the thermodynamic approach of the surface contact phenomena at the level of the eyeball are currently mechanisms for understanding and determining the functioning of the visual process. All these results support the need to build complex and modularly connected models through which the user can implement particular cases and obtain answers to different situations. Thus, in the first part of the paper are presented, synthetically, aspects related to normal eye movements and visual dysfunctions that can introduce spatial and temporal limitations of binocular balance. The approach from the point of view of the Newtonian mechanics of the synergistic movement of the eyeballs can be considered efficient for obtaining the laws of motion of the eyeball in relation to its connections in the structure of the elastic supporting elements. The second part of the paper focuses on the implementation of a methodology for analyzing eye movements by developing an eyeball model based on the principles of mechanics and considered as a solid with a fixed point and which is operated with elastic and damping elements. The third part of the paper presents the observations and conclusions on the development of this model for exemplifying eye movements. This model is found to be necessary for understanding the set of information to be transformed by the retinal sensory surface and transmitted to the visual cortex for analysis and processing.*

Keywords: ocular movements, modeling, biomechanics.

Introduction

The analysis models of human sensory systems have always been mechanisms for understanding the functioning and development of relationships of these structures with others in the human body or with the environment. Each time, these models and quantifications of behaviour seek to establish a model as complex and complete as possible to achieve as many situations as possible of normal and / or abnormal functioning. In this sense, in a series of theoretical and experimental researches were approached important and essential aspects of eye movements (Polpitiyay, A., 2004, Paraskevoudi, N., 2019, Koene, A., 2002), of the optics of image formation on the

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retina (Wandell, B., 1995), of the perception mechanisms or of the specific physiological processes (Korala, A., 2020), all aiming at knowing and understanding the neuro-motor mechanisms that underlie the whole process of ocular motility. Previous but also recent studies use simple or compound methods of mechanical, optical or neuro-motor modelling to understand especially the control structure of eye movements in their synergy.
In this sense, two directions of analysis were approached, the first based on the properties of the external muscular system of the eyeball and the second focusing on the movement control mechanism (Polpitiya, A., 2007, Martin, C., 1998, Miller, J., 1984) and using simplified, linear models focused only on information processing and control aspects and less on the final results. In order to achieve such methodologies for modelling eye movements, a series of simplifying hypotheses are required, as follows: first, the space in which eye movement takes place is defined by two systems of tri-orthogonal axes, one fixed on the cranial orbit and one mobile, originating in the centre of rotation of the eyeball.
Secondly, the eyeball is assimilated with a perfect sphere and a third hypothesis refers to the fact that all eye movements are subject to the Listing law.
Thus, the dynamics of the eye can be treated as a mechanical system with holonomic constraints, which essentially limits the configuration space to behave at the same time as an integral system and as a two-dimensional subsystem of the whole system visual. As shown in most previous studies on eye movement, it is assumed that the head remained fixed and only the eyeball is allowed to move freely. In addition, Listing pointed out that in this situation the orientation of the eye was completely determined by the direction of its gaze, i.e. the axis of sight. It was later shown that starting from the primary position of the frontal gaze, any other direction of view is obtained by a rotation matrix whose axis of rotation is constrained to overlap with a plane, called the Listing plane. Consequently, the set of all the orientations that the eye can assume is represented by a subsystem of the eyeball called *LIST*.

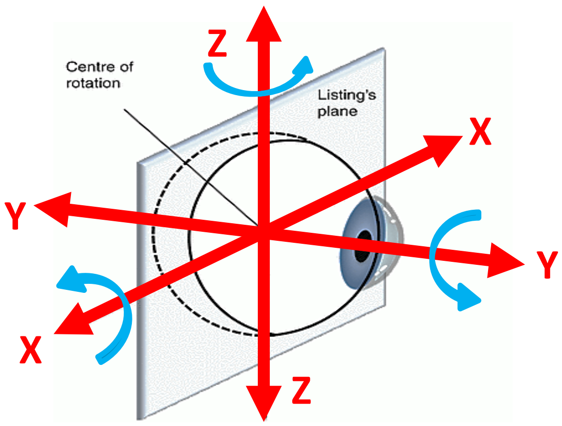

Figure no. 1. Schematic of the eyeball planes and axes - Listing's plane and Fick's axes
Source: [https://entokey.com/movements-of-the-eye/].

Thus, Listing was the one who showed that, in a fixed coordinate system on the head, the orientations of the eyes are restricted to this specific sub-category called LIST. The most interesting problem is the situation in which the eye movement is accompanied by the movement of the head, much wider movement as angular values, but also combined on the three directions of space, thus obtaining a series of movement curves that will have to satisfy the constraints of the Listing plan. (Ghosh, B., 2010)

The binocular movements, governed by the temporal and energetic synergy, are determined to take place in the orbital cavity, through the external muscular system and are coordinated by the neuro-motor system of the cranial nerves (SR = superior rectus, MR = medial rectus, LR = lateral rectus, IR = lower rectus, IO = lower oblique, SO = upper oblique).

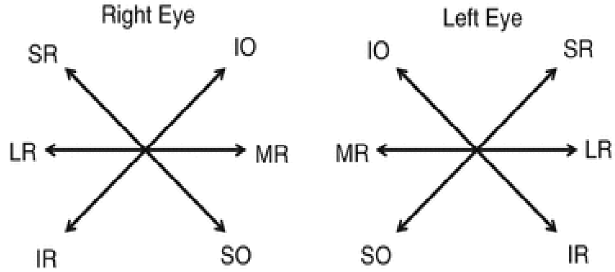

Figure no. 2. Fields of eyeballs action's
Source: [https://entokey.com/movements-of-the-eye/].

As mentioned in various research materials, the antagonistic actions "pull" the eye in opposite directions while the synergistic actions orient the axis of sight of the eyes in the same direction (fig.no.3). In addition, during the movement of the head, two eye stabilization systems work namely the vestibulo-ocular system and the opto-kinetic system, with conjugated movements, in which both eyes move in the same direction. Depending on the type of eye movement, one can also identify the functions of stabilizing the gaze through the procedure of following the fixing target in motion and directing towards it, with the possibility of adjusting the view for different distances (lens adjustment process).

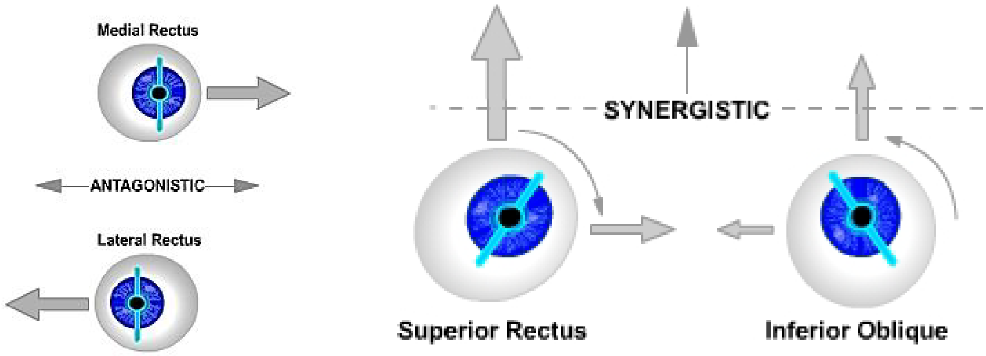

Figure no. 3. The extraocular muscles of the right eye and their actions.
Source: [https://nba.uth.tmc.edu/neuroscience/m/s3/chapter08.html].

Any deviation from the synergy of binocular movement can lead to the appearance of visual dysfunctions such as diplopia, blurred vision, effort and eye fatigue, which, in turn, accumulated and / or non-rehabilitation can create permanent visual problems.

Theoretical substantiation of the eyeball movement model

As presented in the literature, it is useful in the development of analyzes, a representation of the "orientation of the eye" using quaternion in the form of a set of four real numbers that form the set Q. Thus each element can be written q ∈ Q as defined by the following relationship:

$$q = q_0 1 + q_1 i + q_2 j + q_3 k \tag{1}$$

where the term $q_0 1$ = the scalar part, and $q_1 i + q_2 j + q_3 k$ = the vector part.

This vector is defined by three components (q_1, q_2 and q_3) with representation in tri-orthogonal system. Therefore the space of a quaternion unit can be written as given by the following relation:

$$q = cos\frac{\Phi}{2}1 + sin\frac{\phi}{2}n_1 i + sin\frac{\phi}{2}n_2 j + sin\frac{\Phi}{2}n_3 k \qquad (2)$$

where $\phi \in [0;2\pi]$, and $n = (n_1;n_2;n_3)$ represents the unit vector in space R^3

Constructing the unit vector specific to the R^3 space, the authors (Ghosh, B., Wijayasingle, I., 2010) managed to combine the defined relations (2) and (3) obtaining, at the end, a parameterization relation of the quaternion unit (4) by which can then define the rotation matrices in relation to the angles *θ; ϕ; α* and respecting the initial direction of the gaze.

$$n = \begin{pmatrix} cos\theta \quad cos\alpha \\ sin\theta \quad cos\alpha \\ sin\alpha \end{pmatrix} \qquad (3)$$

$$q = \begin{pmatrix} cos\frac{\phi}{2} \\ sin\frac{\phi}{2}\ cos\theta\ cos\alpha \\ sin\frac{\phi}{2}\ sin\theta\ cos\alpha \\ sin\frac{\phi}{2} sin\alpha \end{pmatrix} \qquad (4)$$

For these reasons, Listing constraints are a form of restriction on the axis of rotation, especially for the situation when the transition of the gaze direction to a direction beyond the primary direction of gaze is made. Therefore, this transition of the gaze between the secondary position and any tertiary position (much further from the primary direction) will be possible to achieve another axis of rotation in the Listing plane.

Therefore, when the head is assumed to be fixed during the transition between two arbitrary directions of gaze, the Listing law must be considered, at all times, as a constraint on the instantaneous orientations of the eye. But in the situation where the head is allowed to move randomly, then the directions of eye orientation are allowed to "escape" from the space of action of the visual function during the transition between the two points defined by the two directions of gaze another random).

This aspect translates into the fact that during the transfer of the direction of gaze from the secondary position to the tertiary position the eye movement, associated with the head movement can be random, not directed to a point in space and without respecting a quantifiable law of motion.

Approach to eye movement from the point of view of the Euler rotation theorem

As described in Euler's rotation theorem, any rotational motion can be defined by the three angles (Φ, θ, Ψ) formed by the axes of a fixed reference system compared to a mobile one (fig.no.4).

These angles define the rotational movements around the axes of the fixed reference system, as follows: the first rotation is at an angle Φ, around the axis *z*, the second rotation is at an angle θ, performed around the *x*-axis, and the third rotation is performed at an angle Ψ.

The rotation matrix (A) is defined as a scalar product between the matrices that define each type of rotation, in the range of specific angles (B, C, D).

The components of angular velocities can be obtained from the development of these matrices ω, related to the axes of the body (eyeball). Obviously, the shape of these parameters will also be written with the help of a matrix in relation to the Euler angles (Φ, θ, Ψ).

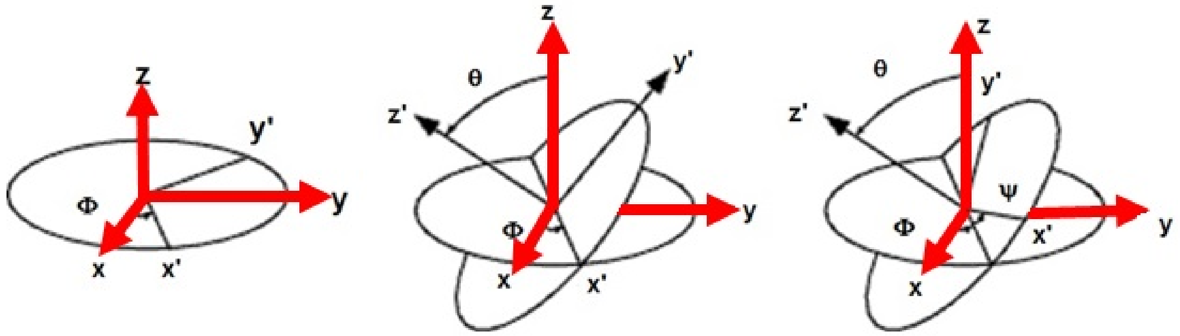

Figure no. 4. Euler's angles
Source: [https://mathworld.wolfram.com/EulerAngles.html].

Therefore, for the analysis of the rotational motion of the eyeball, if the coordinates of two sets of points are known (an initial position and a rotated one with respect to it) then Euler's rotational matrix can be obtained by a simple procedure, using the method *least squares fitting*.

$$B = \begin{vmatrix} cos\Psi & sin\Psi & 0 \\ -sin\Psi & cos\Psi & 0 \\ 0 & 0 & 1 \end{vmatrix}; \quad C = \begin{vmatrix} 1 & 0 & 0 \\ 0 & cos\theta & sin\theta \\ 0 & -sin\theta & cos\theta \end{vmatrix}; \quad D = \begin{vmatrix} cos\Phi & sin\Phi & 0 \\ -sin\Phi & cos\Phi & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{5}$$

$$A = B\ C\ D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \tag{6}$$

Thus the vector matrix becomes:

$$[x'_1 \dots \dots \dots x'_n] = A[x_1 \dots \dots \dots . x_n] \tag{7}$$

and from solving this equation the matrix A is obtained as:

$$A = X'X^T(XX^T)^{-1} \tag{8}$$

This matrix is then customized in relation to the type of connections that exist between the components of the eyeball and the anatomical structures in the skull-frontal area (muscles, tendons and orbit) to obtain the trajectory of movement in the functioning of the visual system. If they are deficient in the movements performed by the eyeball to orient the aiming axis towards the fixation point, then they are determined by the limited values of the angular fields, respectively by the connections between the reference axes.

Results and discussions

The equations for substantiating the movements of the eyeball during the process of orientation of the aiming axis towards the fixation point represent a theoretical approach that highlights the need to permanently know the position of the central fovea (the end point of the aiming axis). Of all the models studied, the closest to reality is identified as the model of the eyeball as a solid with a fixed point of rotation, but also for it there are simplifying hypotheses and limiting requirements. Through this model it was found that regardless of the position of the head, the orientation of the visual axis towards the fixation point does not change as long as the image is formed in the fovea, even in the conditions of rotation of the eyeball.

Conclusion

Therefore, the analysis models of the movements of the eyeball, seen through the prism of the laws of Newtonian mechanics, represent the most efficient approach of the kinematic analyses of positioning and orientation in the vision process. It can therefore be concluded that during the formation of the image on the photosensitive surface which is represented by the retina, that the limited (angular) movements of the eyeball do not change the way of forming the final image in monocular and binocular system, respectively.

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