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DYNAMIC MODELING FOR TWO ROBOTS IN COOPERATIVE MOVEMENTS 3TR-2R

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Abstract: *In this paper, the dynamic modeling for a cooperative structure (3TR-2R), will be developed. The classical iterative algorithm was applied to determine the dynamic equations. In this algorithm we use on the one hand the Newton-Euler type equations and on the other hand the Lagrange-Euler type equations. Knowing the parameters of the mass distribution will determine: the speeds and accelerations associated with the centers of mass, the external forces and moments, the connecting forces and moments and the generalized driving forces.*
Keywords: *cooperation, advanced mechanics, dynamic modeling.*

INTRODUCTION

Fundamental theorems, in the dynamics of mechanical systems from which the mechanical structures of robots are analyzed, play an essential role in determining the matrix equations of dynamics or dynamic control functions. These theorems are based on the fundamental notions of the dynamics of mechanical systems, among which are: acceleration energy, kinetic energy, mechanical work, kinetic moment, impulse.

2. Equations of inverse dynamics for structure, 3TR

Mass distribution
The mass distribution parameters are included in Table 1

Table 1 The mass distribution parameters

Elements i	Mass M_i	Center of mass ${}^{i}\bar{r}_{Ci}$	Inertial Tenor ${}^{i}J_{i}^{*}$

1	M_1	${}^{1}\bar{r}_{C1} = \begin{bmatrix} 0 \\ 0 \\ l_1/2 \end{bmatrix}$	${}^{1}I_1^{*} = \begin{bmatrix} {}^{1}I_x^{*} & 0 & 0 \\ 0 & {}^{1}I_y^{*} & 0 \\ 0 & 0 & {}^{1}I_z^{*} \end{bmatrix}$
2	M_2	${}^{2}\bar{r}_{C2} = \begin{bmatrix} l_2/2 \\ 0 \\ 0 \end{bmatrix}$	${}^{2}I_2^{*} = \begin{bmatrix} {}^{2}I_x^{*} & 0 & 0 \\ 0 & {}^{2}I_y^{*} & 0 \\ 0 & 0 & {}^{2}I_z^{*} \end{bmatrix}$
3	M_3	${}^{3}\bar{r}_{C3} = \begin{bmatrix} 0 \\ 0 \\ d_3/2 \end{bmatrix}$	${}^{3}I_3^{*} = \begin{bmatrix} {}^{3}I_x^{*} & 0 & 0 \\ 0 & {}^{3}I_y^{*} & 0 \\ 0 & 0 & {}^{3}I_z^{*} \end{bmatrix}$
4	M_4	${}^{4}\bar{r}_{C4} = \begin{bmatrix} 0 \\ 0 \\ d_4/2 \end{bmatrix}$	${}^{4}I_4^{*} = \begin{bmatrix} {}^{4}I_x^{*} & 0 & 0 \\ 0 & {}^{4}I_y^{*} & 0 \\ 0 & 0 & {}^{4}I_z^{*} \end{bmatrix}$

The payload to be handled is defined by the following moment force vectors:

$$ {}^{5}\bar{f}_5 = \begin{bmatrix} {}^{5}f_x \\ {}^{5}f_y \\ {}^{5}f_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ {}^{5}\bar{n}_5 = \begin{bmatrix} {}^{5}n_x \\ {}^{5}n_y \\ {}^{5}n_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{1} $$

Speeds corresponding to mass centers:

$$ {}^{1}\bar{v}_{C_1} = \begin{bmatrix} 0 \\ \dot{q}_1 \\ 0 \end{bmatrix};\ {}^{2}\bar{v}_{C_2} = \begin{bmatrix} 0 \\ \dot{q}_1 \\ -\dot{q}_2 \end{bmatrix};\ {}^{3}\bar{v}_{C_3} = \begin{bmatrix} \dot{q}_3 \\ \dot{q}_1 \\ -\dot{q}_2 \end{bmatrix};\ {}^{4}\bar{v}_{C_4} = \begin{bmatrix} sq_4 \cdot \dot{q}_1 + cq_4 \cdot \dot{q}_3 \\ cq_4 \cdot \dot{q}_1 - sq_4 \cdot \dot{q}_3 \\ -\dot{q}_2 \end{bmatrix} \tag{2} $$

Accelerations corresponding to centers of mass:

$$ {}^{1}\dot{\bar{v}}_{C_1} = {}^{1}\dot{\bar{v}}_1 = \begin{bmatrix} 0 \\ \ddot{q}_1 \\ 0 \end{bmatrix};\ {}^{2}\dot{\bar{v}}_{C_2} = {}^{2}\dot{\bar{v}}_2 = \begin{bmatrix} 0 \\ \ddot{q}_1 \\ -\ddot{q}_2 \end{bmatrix};\ {}^{3}\dot{\bar{v}}_{C_3} = {}^{3}\dot{\bar{v}}_3 = \begin{bmatrix} \ddot{q}_3 \\ \ddot{q}_1 \\ -\ddot{q}_2 \end{bmatrix};\ {}^{4}\dot{\bar{v}}_{C_4} = {}^{4}\dot{\bar{v}}_4 = \begin{bmatrix} sq_4 \cdot \ddot{q}_1 + cq_4 \cdot \ddot{q}_3 \\ cq_4 \cdot \ddot{q}_1 - sq_4 \cdot \ddot{q}_3 \\ -\ddot{q}_2 \end{bmatrix} \tag{3} $$

External forces for the 3TR structure

$$ {}^{i}\bar{F}_i^{*} = M_i \cdot {}^{i}\dot{\bar{v}}_{C_i};\ {}^{1}\bar{F}_1^{*} = \begin{bmatrix} 0 \\ M_1\ddot{q}_1 \\ 0 \end{bmatrix};\ {}^{2}\bar{F}_2^{*} = \begin{bmatrix} 0 \\ M_2\ddot{q}_1 \\ -M_2\ddot{q}_2 \end{bmatrix};\ {}^{3}\bar{F}_3^{*} = \begin{bmatrix} M_3\ddot{q}_3 \\ M_3\ddot{q}_1 \\ -M_3\ddot{q}_2 \end{bmatrix};\ {}^{4}\bar{F}_4^{*} = \begin{bmatrix} M_4 \cdot sq_4 \cdot \ddot{q}_1 + M_4 \cdot cq_4 \cdot \ddot{q}_3 \\ M_4 \cdot cq_4 \cdot \ddot{q}_1 - M_4 \cdot sq_4 \cdot \ddot{q}_3 \\ -M_4\ddot{q}_2 \end{bmatrix} \tag{4} $$

Moments of the External Forces for the 3TR structure

$$ {}^{1}\bar{N}_1^{*} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ {}^{2}\bar{N}_2^{*} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ {}^{3}\bar{N}_3^{*} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ {}^{4}\bar{N}_4^{*} = \begin{bmatrix} 0 \\ 0 \\ {}^{4}I_z^{*} \cdot \ddot{q}_4 \end{bmatrix} \tag{5} $$

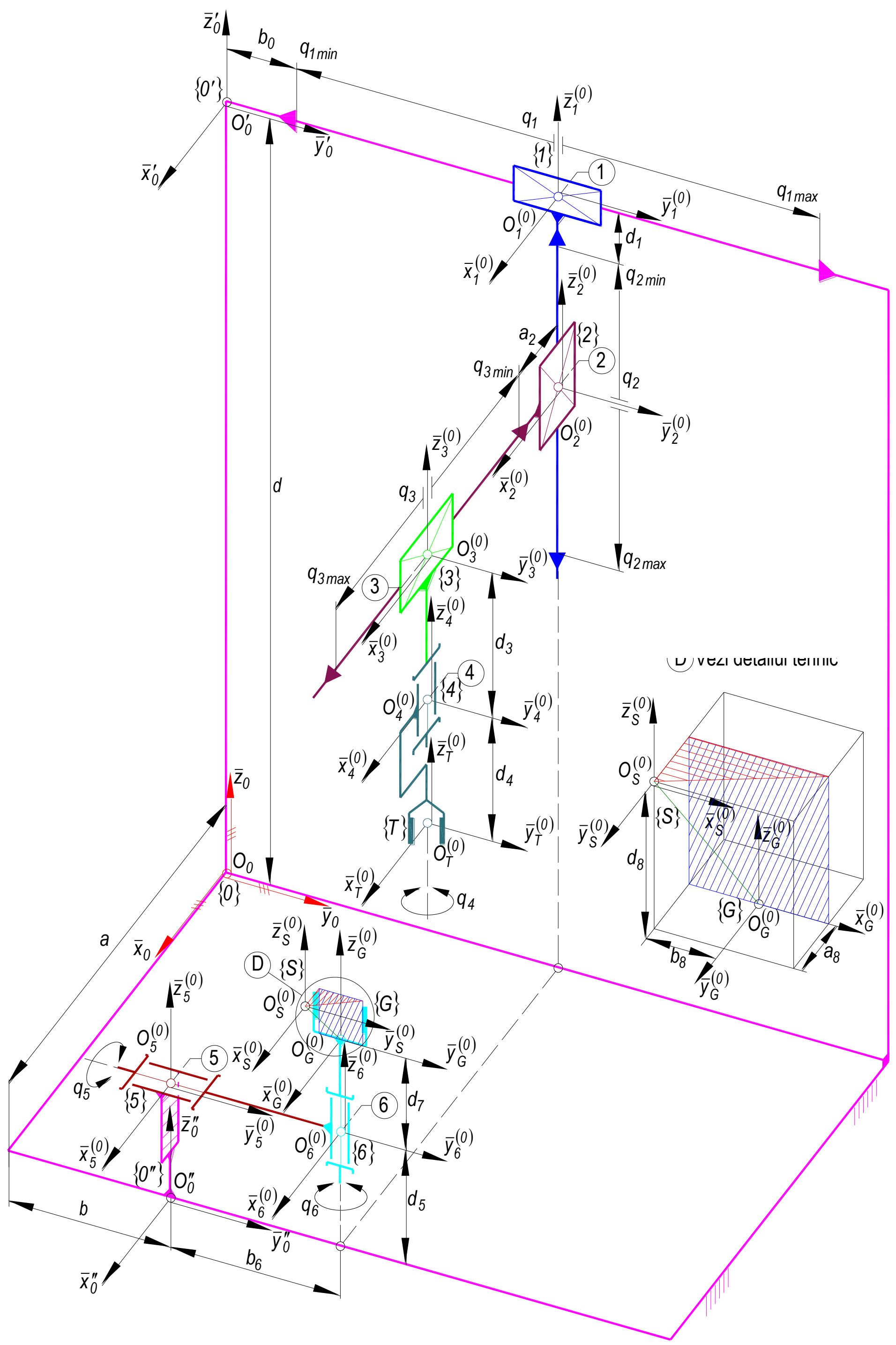

Figure 1: Kinematic scheme of the mechanical structure at zero configuration 3TR-2R.

The binding forces for the 3TR structure

$$
{}^{4}\overline{f}_4 = \begin{bmatrix} M_4 \cdot sq_4 \cdot \ddot{q}_1 + M_4 \cdot cq_4 \cdot \ddot{q}_3 \\ M_4 \cdot cq_4 \cdot \ddot{q}_1 - M_4 \cdot sq_4 \cdot \ddot{q}_3 \\ -M_4 \ddot{q}_2 \end{bmatrix};\ {}^{3}\overline{f}_3 = \begin{bmatrix} M_3 \ddot{q}_3 + M_4 \cdot \ddot{q}_3 \\ M_4 \cdot \ddot{q}_1 + M_3 \ddot{q}_1 \\ -M_4 \ddot{q}_2 - M_3 \ddot{q}_2 \end{bmatrix};\ {}^{2}\overline{f}_2 = \begin{bmatrix} M_3 \ddot{q}_3 + M_4 \cdot \ddot{q}_3 \\ M_4 \cdot \ddot{q}_1 + M_3 \ddot{q}_1 + M_2 \ddot{q}_1 \\ -M_4 \ddot{q}_2 - M_3 \ddot{q}_2 - M_2 \ddot{q}_2 \end{bmatrix};
$$

$$
{}^{1}\overline{f}_1 = \begin{bmatrix} M_3 \ddot{q}_3 + M_4 \cdot \ddot{q}_3 \\ M_4 \cdot \ddot{q}_1 + M_3 \ddot{q}_1 + M_2 \ddot{q}_1 + M_1 \ddot{q}_1 \\ -M_4 \ddot{q}_2 - M_3 \ddot{q}_2 - M_2 \ddot{q}_2 \end{bmatrix} \tag{6}
$$

The moments of the connecting forces for the 3TR structure

$$
{}^{4}\overline{n}_4 = \begin{bmatrix} 0 \\ 0 \\ {}^{4}I_z^* \cdot \ddot{q}_4 \end{bmatrix};\ {}^{3}\overline{n}_3 = \begin{bmatrix} -d_3 \cdot M_4 \cdot \ddot{q}_1 \\ -d_3 \cdot M_4 \cdot \ddot{q}_3 \\ {}^{4}I_z^* \cdot \ddot{q}_4 \end{bmatrix};\ {}^{2}\overline{n}_2 = \begin{bmatrix} -d_3 \cdot M_4 \cdot \ddot{q}_1 \\ \hline -d_3 \cdot M_4 \cdot \ddot{q}_3 - a_2 \cdot M_4 \ddot{q}_2 - \\ -a_2 \cdot M_3 \ddot{q}_2 - q_3 \cdot M_4 \ddot{q}_2 - q_3 \cdot M_3 \ddot{q}_2 \\ \hline {}^{4}I_z^* \cdot \ddot{q}_4 - a_2 \cdot M_4 \cdot \ddot{q}_1 - \\ -a_2 \cdot M_3 \ddot{q}_1 - q_3 \cdot M_4 \cdot \ddot{q}_1 - q_3 \cdot M_3 \ddot{q}_1 \end{bmatrix} \tag{7}
$$

$$
{}^{1}\overline{n}_1 = \begin{bmatrix} -d_3 \cdot M_4 \cdot \ddot{q}_1 - d_1 \cdot M_4 \cdot \ddot{q}_1 - d_1 \cdot M_3 \ddot{q}_1 - d_1 \cdot M_2 \ddot{q}_1 - q_2 \cdot M_4 \cdot \ddot{q}_1 - q_2 \cdot M_3 \ddot{q}_1 - q_2 \cdot M_2 \ddot{q}_1 \\ -d_3 \cdot M_4 \cdot \ddot{q}_3 - a_2 \cdot M_4 \ddot{q}_2 - a_2 \cdot M_3 \ddot{q}_2 - q_3 \cdot M_4 \ddot{q}_2 - q_3 \cdot M_3 \ddot{q}_2 + d_1 \cdot M_3 \ddot{q}_3 + d_1 \cdot M_4 \cdot \ddot{q}_3 + \\ + q_2 \cdot M_3 \ddot{q}_3 + q_2 \cdot M_4 \cdot \ddot{q}_3 \\ {}^{4}I_z^* \cdot \ddot{q}_4 - a_2 \cdot M_4 \cdot \ddot{q}_1 - a_2 \cdot M_3 \ddot{q}_1 - q_3 \cdot M_4 \cdot \ddot{q}_1 - q_3 \cdot M_3 \ddot{q}_1 \end{bmatrix} \tag{8}
$$

The generalized driving forces for the 3TR structure are determined as follows:

	$\left(\overline{k}_1^{(0)} \equiv \overline{y}_1^{(0)};\ \overline{k}_2^{(0)} \equiv \overline{z}_2^{(0)};\ \overline{k}_3^{(0)} \equiv \overline{x}_3^{(0)};\ \overline{k}_4^{(0)} \equiv \overline{z}_4^{(0)}\right)$	(9)
	$Q_m^1 = \left\{ {}^{1}\overline{f}_1^{T} \cdot \left(1 - \Delta_1\right) + {}^{1}\overline{n}_1^{T} \cdot \Delta_1 \right\} \cdot {}^{1}\overline{k}_1 = \begin{bmatrix} {}^{1}f_x & {}^{1}f_y & {}^{1}f_z \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = {}^{1}f_y = M_3 \ddot{q}_1 + M_2 \ddot{q}_1 + M_1 \ddot{q}_1$	(10)
	$Q_m^2 = \left\{ {}^{2}\overline{f}_2^{T} \cdot \left(1 - \Delta_2\right) + {}^{2}\overline{n}_2^{T} \cdot \Delta_2 \right\} \cdot {}^{2}\overline{k}_2 = \begin{bmatrix} {}^{2}f_x & {}^{2}f_y & {}^{2}f_z \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = {}^{2}f_z = -M_4 \ddot{q}_2 - M_3 \ddot{q}_2 - M_2 \ddot{q}_2$	(11)
	$Q_m^3 = \left\{ {}^{3}\overline{f}_3^{T} \cdot \left(1 - \Delta_3\right) + {}^{3}\overline{n}_3^{T} \cdot \Delta_3 \right\} \cdot {}^{3}\overline{k}_3 = \begin{bmatrix} {}^{3}f_x & {}^{3}f_y & {}^{3}f_z \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = {}^{3}f_x = {}^{3}f_x = M_3 \ddot{q}_3 + M_4 \cdot \ddot{q}_3$	(12)
	$Q_m^4 = \left\{ {}^{4}\overline{f}_4^{T} \cdot \left(1 - \Delta_i\right) + {}^{4}\overline{n}_4^{T} \cdot \Delta_4 \right\} \cdot {}^{4}\overline{k}_4 = \begin{bmatrix} {}^{4}n_x & {}^{4}n_y & {}^{4}n_z \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = {}^{4}n_z = {}^{4}I_z^* \cdot \ddot{q}_4 + {}^{4}I_z^* \dot{q}_4$	(13)

2.Equations of inverse dynamics for structure, 2R

The mass distribution parameters are included in Table 2

Table 2

Elements i	Mass M_i	Center of mass ${}^{i}\overline{r}_{Ci}$	Inertial Tensor ${}^{i}J_i^*$
5	M_5	${}^{5}\overline{r}_{C_5} = \begin{bmatrix} 0 \\ b_6 / 2 \\ 0 \end{bmatrix}$	${}^{5}I_5^* = \begin{bmatrix} {}^{5}I_x^* & 0 & 0 \\ 0 & {}^{5}I_y^* & 0 \\ 0 & 0 & {}^{5}I_z^* \end{bmatrix}$

6	M_6	${}^{6}\bar{r}_{C_6}=\begin{bmatrix}0\\0\\d_7/2\end{bmatrix}$	${}^{6}I_6^{*}=\begin{bmatrix}{}^{6}I_x^{*} & 0 & 0\\0 & {}^{6}I_y^{*} & 0\\0 & 0 & {}^{6}I_z^{*}\end{bmatrix}$

The payload to be handled is defined by the following moment force vectors:

$$ {}^{7}\bar{f}_7=\begin{bmatrix}{}^{7}f_x\\{}^{7}f_y\\{}^{7}f_z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix};\ {}^{7}\bar{n}_7=\begin{bmatrix}{}^{7}n_x\\{}^{7}n_y\\{}^{7}n_z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \tag{14}$$

Speeds corresponding to mass centers:

$$ {}^{5}\bar{v}_{C_5}=\begin{bmatrix}0\\0\\0\end{bmatrix};\ {}^{6}\bar{v}_{C_6}=\begin{bmatrix}\dot{q}_5\cdot cq_6\cdot d_7/2\\-\dot{q}_5\cdot sq_6\cdot d_7/2\\0\end{bmatrix} \tag{15}$$

Accelerations corresponding to centers of mass:

$$ {}^{5}\dot{\bar{v}}_{C_5}=\begin{bmatrix}0\\0\\0\end{bmatrix};\ {}^{6}\dot{\bar{v}}_{C_6}=\begin{bmatrix}\ddot{q}_5\cdot cq_6-\dot{q}_5\cdot\dot{q}_6\cdot sq_6\cdot d_7/2\\-\ddot{q}_5\cdot sq_6+\dot{q}_5\cdot\dot{q}_6\cdot cq_6\cdot d_7/2\\0\end{bmatrix}; \tag{16}$$

External forces for the 3TR structure

$$ {}^{5}\bar{F}_5^{*}=\begin{bmatrix}0\\0\\0\end{bmatrix};{}^{6}\bar{F}_6^{*}=\begin{bmatrix}(M_6\cdot\ddot{q}_5\cdot cq_6-M_6\cdot\dot{q}_5\cdot\dot{q}_6\cdot sq_6\cdot d_7)/2\\(-\ddot{q}_5\cdot M_6\cdot sq_6+M_6\cdot\dot{q}_5\cdot\dot{q}_6\cdot cq_6\cdot d_7)/2\\0\end{bmatrix} \tag{17}$$

Moments of the External Forces for the 3TR structure

$$ {}^{5}\bar{N}_5^{*}=\begin{bmatrix}0\\{}^{5}I_y^{*}\cdot\ddot{q}_5\\0\end{bmatrix};\ {}^{6}\bar{N}_6^{*}=\begin{bmatrix}{}^{6}I_x^{*}\cdot\ddot{q}_5\cdot sq_6+{}^{6}I_x^{*}\cdot\dot{q}_5\cdot\dot{q}_6\cdot cq_6-\dot{q}_6\cdot{}^{5}I_y^{*}\cdot\dot{q}_5\cdot cq_6+\dot{q}_5\cdot cq_6\cdot\dot{q}_6\\{}^{6}I_y^{*}\cdot\ddot{q}_5\cdot cq_6-{}^{6}I_y^{*}\cdot\dot{q}_5\cdot\dot{q}_6\cdot sq_6+\dot{q}_6\cdot{}^{5}I_x^{*}\cdot\dot{q}_5\cdot sq_6-\dot{q}_5\cdot sq_6\cdot{}^{5}I_z^{*}\cdot\dot{q}_6\\{}^{6}I_y^{*}\cdot\ddot{q}_6-\dot{q}_5\cdot cq_6\cdot{}^{5}I_x^{*}\cdot\dot{q}_5\cdot sq_6+\dot{q}_5\cdot sq_6\cdot{}^{5}I_y^{*}\cdot\dot{q}_5\cdot cq_6\end{bmatrix} \tag{18}$$

The binding forces for the 3TR structure

$$ {}^{6}\bar{f}_6=\begin{bmatrix}0\\0\\0\end{bmatrix};\ {}^{5}\bar{f}_5=\begin{bmatrix}0\\{}^{5}I_y^{*}\cdot\ddot{q}_5\\0\end{bmatrix} \tag{19}$$

The moments of the connecting forces for the 3TR structure

$$ {}^{6}\bar{n}_6=\begin{bmatrix}{}^{6}I_x^{*}\cdot\ddot{q}_5\cdot sq_6+{}^{6}I_x^{*}\cdot\dot{q}_5\cdot\dot{q}_6\cdot cq_6-\dot{q}_6\cdot{}^{5}I_y^{*}\cdot\dot{q}_5\cdot cq_6+\dot{q}_5\cdot cq_6\cdot\dot{q}_6\\{}^{6}I_y^{*}\cdot\ddot{q}_5\cdot cq_6-{}^{6}I_y^{*}\cdot\dot{q}_5\cdot\dot{q}_6\cdot sq_6+\dot{q}_6\cdot{}^{5}I_x^{*}\cdot\dot{q}_5\cdot sq_6-\dot{q}_5\cdot sq_6\cdot{}^{5}I_z^{*}\cdot\dot{q}_6\\{}^{6}I_y^{*}\cdot\ddot{q}_6-\dot{q}_5\cdot cq_6\cdot{}^{5}I_x^{*}\cdot\dot{q}_5\cdot sq_6+\dot{q}_5\cdot sq_6\cdot{}^{5}I_y^{*}\cdot\dot{q}_5\cdot cq_6\end{bmatrix} \tag{20}$$

$$ {}^{5}\bar{n}_5=\begin{bmatrix}cq_6\cdot{}^{6}I_x^{*}\cdot\ddot{q}_5\cdot sq_6+cq_6\cdot{}^{6}I_x^{*}\cdot\dot{q}_5\cdot\dot{q}_6\cdot cq_6-cq_6\cdot\dot{q}_6\cdot{}^{5}I_y^{*}\cdot\dot{q}_5\cdot cq_6+cq_6\cdot\dot{q}_5\cdot cq_6\cdot\dot{q}_6-\\-sq_5\cdot{}^{6}I_y^{*}\cdot\ddot{q}_5\cdot cq_6+sq_5\cdot{}^{6}I_y^{*}\cdot\dot{q}_5\cdot\dot{q}_6\cdot sq_6-sq_5\cdot\dot{q}_6\cdot{}^{5}I_x^{*}\cdot\dot{q}_5\cdot sq_6+sq_5\cdot\dot{q}_5\cdot sq_6\cdot{}^{5}I_z^{*}\cdot\dot{q}_6\\sq_6\cdot{}^{6}I_x^{*}\cdot\ddot{q}_5\cdot sq_6+sq_6\cdot{}^{6}I_x^{*}\cdot\dot{q}_5\cdot\dot{q}_6\cdot cq_6-sq_6\cdot\dot{q}_6\cdot{}^{5}I_y^{*}\cdot\dot{q}_5\cdot cq_6+sq_6\cdot\dot{q}_5\cdot cq_6\cdot\dot{q}_6+\\+cq_6\cdot{}^{6}I_y^{*}\cdot\ddot{q}_5\cdot cq_6-cq_6\cdot{}^{6}I_y^{*}\cdot\dot{q}_5\cdot\dot{q}_6\cdot sq_6+cq_6\cdot\dot{q}_6\cdot{}^{5}I_x^{*}\cdot\dot{q}_5\cdot sq_6-cq_6\cdot\dot{q}_5\cdot sq_6\cdot{}^{5}I_z^{*}\cdot\dot{q}_6+{}^{5}I_y^{*}\cdot\ddot{q}_5\\0\end{bmatrix} \tag{21}$$

The generalized driving forces for the 2R structure are determined as it follows:

$$Q_m^5 = q_6 \cdot {}^6I_x^* \cdot \ddot{q}_5 \cdot sq_6 + sq_6 \cdot {}^6I_x^* \cdot \dot{q}_5 \cdot \dot{q}_6 \cdot cq_6 - sq_6 \cdot \dot{q}_6 \cdot {}^5I_y^* \cdot \dot{q}_5 \cdot cq_6 + sq_6 \cdot \dot{q}_5 \cdot cq_6 \cdot \dot{q}_6 + \\ + cq_6 \cdot {}^6I_y^* \cdot \ddot{q}_5 \cdot cq_6 - cq_6 \cdot {}^6I_y^* \cdot \dot{q}_5 \cdot \dot{q}_6 \cdot sq_6 + cq_6 \cdot \dot{q}_6 \cdot {}^5I_x^* \cdot \dot{q}_5 \cdot sq_6 - cq_6 \cdot \dot{q}_5 \cdot sq_6 \cdot {}^5I_z^* \cdot \dot{q}_6 + {}^5I_y^* \cdot \ddot{q}_5 \quad (22)$$

$$Q_m^6 = {}^6I_y^* \cdot \ddot{q}_6 - \dot{q}_5 \cdot cq_6 \cdot {}^5I_x^* \cdot \dot{q}_5 \cdot sq_6 + \dot{q}_5 \cdot sq_6 \cdot {}^5I_y^* \cdot \dot{q}_5 \cdot cq_6 \quad (23)$$

3.CONCLUSIONS

In this paper, the dynamic modeling for a cooperative structure (3TR-2R), will be developed. The classical iterative algorithm was applied to determine the dynamic equations. In this algorithm we use on the one hand the Newton-Euler type equations and on the other hand the Lagrange-Euler type equations. Knowing the parameters of the mass distribution will determine: the speeds and accelerations associated with the centers of mass, the external forces and moments, the connecting forces and moments and the generalized driving forces.

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