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INVERSE DYNAMIC MODELING OF THE 3R ROBOT USING THE SYMBOLIC COMPUTATION IN MATLAB

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Abstract: *This paper presents mass geometry specific parameters for the 3R robot structure for each element, the absolute linear velocity of the mass center and the absolute linear acceleration of the mass center, the external force, the moment of the constraint force, the constraint force, the moment of the constraint force and the generalized driving forces. An important part of modelling and simulation of a robot is the generation of the geometric, kinematic and dynamic model of its mechanical structure. The software module presented in this paper allows the modelling of the generalized mechanical structure of the 3R robot and the automatic generation of the dynamic model equations of the considered robot. The application is written in MATLAB.*

Keywords: *symbolic computation, 3R structure, constraint force*

INTRODUCTION

The dynamic modelling of a robot is an important step in modelling the mechanical structure of the robot to be analyzed. The symbolic computation deals with symbolic objects, the input data being symbolic and, sometimes, numeric, while the output data are algebraic expressions. The main advantage of the symbolic computation in the robot dynamic modelling consists in direct generation of the symbolic form of the equations of the generalized forces needed to ensure the position, the linear and angular velocity and acceleration of the end-effector, considering the action of the dynamic parameters such as mass properties: masses, mass centers, moments of inertia as well as the payload. The obtained data are useful to generate the simulation of the dynamic behavior of the considered robot.

1.Equations of inverse dynamics for structure, 3R

Mass distribution

The position vector of the mass center, in the frame $\{i\}$:

$$\bar{r}_{C2}^{2} = \begin{bmatrix} x^{2}_{c2} \\ y^{2}_{c2} \\ z^{2}_{c2} \end{bmatrix}; \qquad \bar{r}_{C3}^{3} = \begin{pmatrix} x^{3}_{C3} \\ y^{3}_{C3} \\ z^{3}_{C3} \end{pmatrix} \tag{1}$$

The inertia tensor, axial and centrifugal with respect to the frame $\{i^*\}$:

$$I_1^{1s} = \begin{pmatrix} I_x^{1s} & -I_{xy}^{1s} & -I_{xz}^{1s} \\ -I_{yx}^{1s} & I_y^{1s} & -I_{yz}^{1s} \\ -I_{zx}^{1s} & -I_{zy}^{1s} & I_z^{1s} \end{pmatrix}; \; I_2^{2s} = \begin{pmatrix} I_x^{2s} & -I_{xy}^{2s} & -I_{xz}^{2s} \\ -I_{yx}^{2s} & I_y^{2s} & -I_{yz}^{2s} \\ -I_{zx}^{2s} & -I_{zy}^{2s} & I_z^{2s} \end{pmatrix}; \; I_3^{3s} = \begin{pmatrix} I_x^{3s} & -I_{xy}^{3s} & -I_{xz}^{3s} \\ -I_{yx}^{3s} & I_y^{3s} & -I_{yz}^{3s} \\ -I_{zx}^{3s} & -I_{zy}^{3s} & I_z^{3s} \end{pmatrix} \tag{2}$$

The absolute linear velocity of the mass center:

$$\bar{v}_{C1}^{1} = \begin{pmatrix} \dot{q}_1 z_{C1}^{1} \\ 0 \\ -\dot{q}_1 x_{C1}^{1} \end{pmatrix}; \quad \bar{v}_{C2}^{2} = \begin{pmatrix} \dot{q}_2 + \dot{q}_1 z_{C2}^{2}\,\mathrm{c}(q_2) + \dot{q}_1 y_{C2}^{2}\,\mathrm{s}(q_2) \\ -\dot{q}_2 z_{C2}^{2} - l_1 \dot{q}_1\,\mathrm{s}(q_2) - \dot{q}_1 x_{C2}^{2}\,\mathrm{s}(q_2) \\ \dot{q}_2 y_{C2}^{2} - l_1 \dot{q}_1\,\mathrm{c}(q_2) - \dot{q}_1 x_{C2}^{2}\,\mathrm{c}(q_2) \end{pmatrix}$$

$$\bar{v}_{C3}^{3} = \begin{pmatrix} \mathrm{c}(q_3)\sigma_3 - y_{C3}^{3}\sigma_4 - z_{C3}^{3}\sigma_1 - l_1\dot{q}_1\mathrm{s}(q_2)\mathrm{s}(q_3) \\ x_{C3}^{3}\sigma_4 - \mathrm{s}(q_3)\sigma_3 - z_{C3}^{3}\sigma_2 - l_1\dot{q}_1\mathrm{c}(q_3)\mathrm{s}(q_2) \\ l_2\dot{q}_2 + x_{C3}^{3}\sigma_1 + y_{C3}^{3}\sigma_2 - l_1\dot{q}_1\mathrm{c}(q_2) \end{pmatrix} \tag{3}$$

The absolute linear acceleration of the mass center:

$$\dot{\bar{v}}_{C1}^{1} = \begin{pmatrix} -x_{C1}^{1}\dot{q}_1^{\,2} + \ddot{q}_1 z_{C1}^{1} - g\,\mathrm{s}(q_1) \\ 0 \\ -z_{C1}^{1}\dot{q}_1^{\,2} - \ddot{q}_1 x_{C1}^{1} + g\,\mathrm{c}(q_1) \end{pmatrix}; \tag{4}$$

$$\dot{v}_{C2}^{2} = \begin{pmatrix} \ddot{q}_2 - x_{C2}^2\left(\sigma_3 + \sigma_2\right) - g\,\mathrm{s}\,q_1 + \\ +\ddot{q}_1\, z_{C2}^2\,\mathrm{c}\,q_2 + \ddot{q}_1\, y_{C2}^2\,\mathrm{s}\,q_2 - l_1\,\dot{q}_1{}^2\,\mathrm{c}\,q_1 + \\ +\dot{q}_1\,\dot{q}_2\, y_{C2}^2\,\mathrm{c}\,q_2 - \dot{q}_1\,\dot{q}_2\, z_{C2}^2\,\mathrm{s}\,q_2 \\ \hdashline \mathrm{s}\,q_2\sigma_1 - y_{C2}^2\left(\sigma_2 + \dot{q}_2{}^2\right) - 2\dot{q}_1\,\dot{q}_2\,\mathrm{s}\,q_2 - \\ -\ddot{q}_1\, x_{C2}^2\,\mathrm{s}\,q_2 - \dot{q}_1{}^2\, z_{C2}^2\,\mathrm{c}\,q_2\,\mathrm{s}\,q_2 + \dot{q}_1\,\dot{q}_2\, x_{C2}^2\,\mathrm{c}\,q_2 \\ \hdashline \mathrm{c}\left(q_2\right)\sigma_1 - z_{C2}^2\left(\sigma_3 + \dot{q}_2{}^2\right) - 2\dot{q}_1\,\dot{q}_2\,\mathrm{c}\left(q_2\right) - \\ -\ddot{q}_1\, x_{C2}^2\,\mathrm{c}\left(q_2\right) - \dot{q}_1{}^2\, y_{C2}^2\,\mathrm{c}\left(q_2\right)\mathrm{s}\left(q_2\right) - \dot{q}_1\,\dot{q}_2\, x_{C2}^2\,\mathrm{s}\left(q_2\right) \end{pmatrix}; \tag{5}$$

$$\dot{v}_{C3}^{3} = \begin{pmatrix} y_{C3}^3\,\sigma_5 - \mathrm{s}\left(q_3\right)\sigma_8 - x_{C3}^3\left(\sigma_6{}^2 + \sigma_2{}^2\right) + \mathrm{c}\left(q_3\right)\sigma_1 + \\ +z_{C3}^3\,\sigma_6\,\sigma_3 - z_{C3}^3\,\mathrm{c}\left(q_2\right)\sigma_7 - y_{C3}^3\,\sigma_3\,\sigma_2 \\ \hdashline -y_{C3}^3\left(\sigma_6{}^2 + \sigma_3{}^2\right) - \mathrm{c}\left(q_3\right)\sigma_8 - x_{C3}^3\,\sigma_5 - z_{C3}^3\,\sigma_4 - \\ -\mathrm{s}\left(q_3\right)\sigma_1 - z_{C3}^3\,\sigma_6\,\sigma_2 - x_{C3}^3\,\sigma_3\,\sigma_2 \\ \hdashline y_{C3}^3\,\sigma_4 - z_{C3}^3\left(-\sigma_{11} + \dot{q}_1{}^2 + \dot{q}_2{}^2\right) + \mathrm{c}\left(q_2\right)\sigma_9 - 2\dot{q}_1\,\dot{q}_2\,\mathrm{c}\left(q_2\right) - \\ -l_2\,\mathrm{s}\left(q_2\right)\left(\dot{q}_2{}^2\,\mathrm{c}\left(q_1\right) + \dot{q}_1{}^2\,\mathrm{c}\left(q_2\right)^2 + \dot{q}_1{}^2\,\mathrm{c}\left(q_1\right)\mathrm{c}\left(q_2\right)^2 + \sigma_{10}\right) + \\ +x_{C3}^3\,\sigma_6\,\sigma_3 - y_{C3}^3\,\sigma_6\,\sigma_2 + x_{C3}^3\,\mathrm{c}\left(q_2\right)\sigma_7 \end{pmatrix} \tag{6}$$

External force for 3R structure:

$$\bar{F}_1^{1s} = \begin{pmatrix} -M_1\left(x_{C1}^1\,\dot{q}_1{}^2 - \ddot{q}_1\, z_{C1}^1 + g\,\mathrm{s}\left(q_1\right)\right) \\ 0 \\ -M_1\left(z_{C1}^1\,\dot{q}_1{}^2 + \ddot{q}_1\, x_{C1}^1 - g\,\mathrm{c}\left(q_1\right)\right) \end{pmatrix}; \tag{7}$$

$$\bar{F}_2^{2\text{s}} = \left(\begin{array}{c} M_2 \left(\begin{array}{l} \ddot{q}_2 - x_{\text{C2}}^2 \left(\sigma_3 + \sigma_2 \right) - g\,\text{s}\left(q_1 \right) + \ddot{q}_1 z_{\text{C2}}^2\,\text{c}\left(q_2 \right) + \\ + \ddot{q}_1 y_{\text{C2}}^2\,\text{s}\left(q_2 \right) - l_1 \dot{q}_1^{\,2}\,\text{c}\left(q_1 \right) + \\ \dot{q}_1 \dot{q}_2 y_{\text{C2}}^2\,\text{c}\left(q_2 \right) - \dot{q}_1 \dot{q}_2 z_{\text{C2}}^2\,\text{s}\left(q_2 \right) \end{array} \right) \\ \hdashline -M_2 \left(\begin{array}{l} y_{\text{C2}}^2 \left(\sigma_2 + \dot{q}_2^{\,2} \right) - \text{s}\left(q_2 \right)\sigma_1 + \\ 2\dot{q}_1 \dot{q}_2\,\text{s}\left(q_2 \right) + \ddot{q}_1 x_{\text{C2}}^2\,\text{s}\left(q_2 \right) + \\ \dot{q}_1^{\,2} z_{\text{C2}}^2\,\text{c}\left(q_2 \right)\text{s}\left(q_2 \right) - \dot{q}_1 \dot{q}_2 x_{\text{C2}}^2\,\text{c}\left(q_2 \right) \end{array} \right) \\ \hdashline -M_2 \left(\begin{array}{l} z_{\text{C2}}^2 \left(\sigma_3 + \dot{q}_2^{\,2} \right) - \text{c}\left(q_2 \right)\sigma_1 + \\ 2\dot{q}_1 \dot{q}_2\,\text{c}\left(q_2 \right) + \ddot{q}_1 x_{\text{C2}}^2\,\text{c}\left(q_2 \right) + \\ \dot{q}_1^{\,2} y_{\text{C2}}^2\,\text{c}\left(q_2 \right)\text{s}\left(q_2 \right) + \dot{q}_1 \dot{q}_2 x_{\text{C2}}^2\,\text{s}\left(q_2 \right) \end{array} \right) \end{array} \right) \tag{8}$$

$$\bar{F}_3^{3\text{s}} = \left(\begin{array}{c} -M_3 \left(\begin{array}{l} x_{\text{C3}}^3 \left(\sigma_6^{\,2} + \sigma_2^{\,2} \right) + \text{s}\left(q_3 \right)\sigma_8 - y_{\text{C3}}^3 \sigma_5 - \text{c}\left(q_3 \right)\sigma_1 - \\ -z_{\text{C3}}^3 \sigma_6 \sigma_3 + z_{\text{C3}}^3\,\text{c}\left(q_2 \right)\sigma_7 + y_{\text{C3}}^3 \sigma_3 \sigma_2 \end{array} \right) \\ \hdashline -M_3 \left(\begin{array}{l} y_{\text{C3}}^3 \left(\sigma_6^{\,2} + \sigma_3^{\,2} \right) + \text{c}\left(q_3 \right)\sigma_8 + x_{\text{C3}}^3 \sigma_5 + \\ + z_{\text{C3}}^3 \sigma_4 + \text{s}\left(q_3 \right)\sigma_1 + z_{\text{C3}}^3 \sigma_6 \sigma_2 + x_{\text{C3}}^3 \sigma_3 \sigma_2 \end{array} \right) \\ \hdashline -M_3 \left(\begin{array}{l} z_{\text{C3}}^3 \left(-\sigma_{11} + \dot{q}_1^{\,2} + \dot{q}_2^{\,2} \right) - y_{\text{C3}}^3 \sigma_4 - \text{c}\left(q_2 \right)\sigma_9 + 2\dot{q}_1 \dot{q}_2\,\text{c}\left(q_2 \right) + \\ + l_2\,\text{s}\left(q_2 \right)\left(\dot{q}_2^{\,2}\,\text{c}\left(q_1 \right) + \dot{q}_1^{\,2}\,\text{c}\left(q_2 \right)^2 + \dot{q}_1^{\,2}\,\text{c}\left(q_1 \right)\text{c}\left(q_2 \right)^2 + \sigma_{10} \right) - \\ -x_{\text{C3}}^3 \sigma_6 \sigma_3 + y_{\text{C3}}^3 \sigma_6 \sigma_2 - x_{\text{C3}}^3\,\text{c}\left(q_2 \right)\sigma_7 \end{array} \right) \end{array} \right); \tag{9}$$

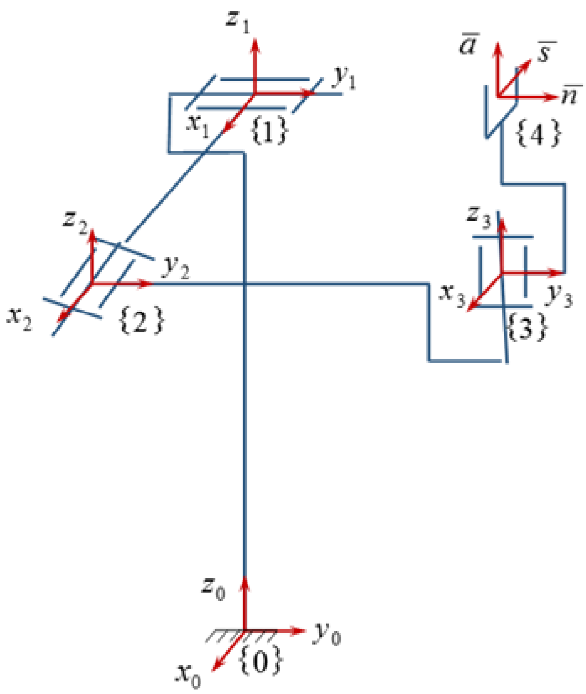

Figure 1: Kinematic scheme of 3R robot

The moment of the constraint force:

$$\bar{N}_1^{1s} = \begin{pmatrix} -I_{zy}^{1s}\,\dot{q}_1^{\,2} - I_{xy}^{1s}\,\ddot{q}_1 \\ I_y^{1s}\,\ddot{q}_1 \\ I_{xy}^{1s}\,\dot{q}_1^{\,2} - I_{zy}^{1s}\,\ddot{q}_1 \end{pmatrix}$$

$$\bar{N}_2^{2s} = \begin{pmatrix} \begin{array}{l} I_{xz}^{2s}\,\ddot{q}_1\,s(q_2) - \dot{q}_1\,s(q_2)\left(I_z^{2s}\,\dot{q}_1\,c(q_2) - I_{yz}^{2s}\,\dot{q}_1\,s(q_2)\right) - \\ -I_{xy}^{2s}\,\ddot{q}_1\,c(q_2) - \dot{q}_1\,c(q_2)\left(I_{zy}^{2s}\,\dot{q}_1\,c(q_2) - I_y^{2s}\,\dot{q}_1\,s(q_2)\right) - \\ -\dot{q}_2\left(I_{zx}^{2s}\,\dot{q}_1\,c(q_2) + I_{yx}^{2s}\,\dot{q}_1\,s(q_2)\right) \\ \hdashline \dot{q}_2\left(I_{zx}^{2s}\,\dot{q}_2 - I_x^{2s}\,\dot{q}_1\,s(q_2)\right) + \dot{q}_1\,c(q_2)\left(I_{zy}^{2s}\,\dot{q}_2 + I_{xy}^{2s}\,\dot{q}_1\,s(q_2)\right) + \\ +\dot{q}_1\,s(q_2)\left(I_z^{2s}\,\dot{q}_2 - I_{xz}^{2s}\,\dot{q}_1\,s(q_2)\right) + I_y^{2s}\,\ddot{q}_1\,c(q_2) + I_{yz}^{2s}\,\ddot{q}_1\,s(q_2) \\ \hdashline \dot{q}_1\,c(q_2)\left(I_y^{2s}\,\dot{q}_2 + I_{xy}^{2s}\,\dot{q}_1\,c(q_2)\right) - \dot{q}_2\left(I_{yx}^{2s}\,\dot{q}_2 + I_x^{2s}\,\dot{q}_1\,c(q_2)\right) + \\ +\dot{q}_1\,s(q_2)\left(I_{yz}^{2s}\,\dot{q}_2 - I_{xz}^{2s}\,\dot{q}_1\,c(q_2)\right) - I_{zy}^{2s}\,\ddot{q}_1\,c(q_2) - I_z^{2s}\,\ddot{q}_1\,s(q_2) \end{array} \end{pmatrix} \tag{10}$$

$$\bar{N}_3^{3s} = \begin{pmatrix} I_{xy}^{3s}\sigma_6 + \sigma_5\left(I_{yz}^{3s}\sigma_5 - I_z^{3s}\sigma_1\right) + \sigma_1\left(I_y^{3s}\sigma_5 - I_{zy}^{3s}\sigma_1\right) + \sigma_2\left(I_{yx}^{3s}\sigma_5 + I_{zx}^{3s}\sigma_1\right) + I_{xz}^{3s}\sigma_4 + I_x^{3s}\sigma_3 \\ \sigma_2\left(I_x^{3s}\sigma_5 + I_{zx}^{3s}\sigma_2\right) - \sigma_5\left(I_{xz}^{3s}\sigma_5 + I_z^{3s}\sigma_2\right) - I_y^{3s}\sigma_6 + \sigma_1\left(I_{xy}^{3s}\sigma_5 - I_{zy}^{3s}\sigma_2\right) + I_{yz}^{3s}\sigma_4 - I_{yx}^{3s}\sigma_3 \\ I_{zy}^{3s}\sigma_6 - \sigma_5\left(I_{xz}^{3s}\sigma_1 + I_{yz}^{3s}\sigma_2\right) + \sigma_2\left(I_x^{3s}\sigma_1 - I_{yx}^{3s}\sigma_2\right) + \sigma_1\left(I_{xy}^{3s}\sigma_1 - I_y^{3s}\sigma_2\right) - I_z^{3s}\sigma_4 - I_{zx}^{3s}\sigma_3 \end{pmatrix}$$

The constraint force:

$$
\overline{f}_3^3 = \begin{pmatrix}
f_{4\mathrm{y}}^4 - M_3 \begin{pmatrix} x_{\mathrm{C3}}^3\left(\sigma_6{}^2 + \sigma_2{}^2\right) + \mathrm{s}(q_3)\sigma_8 - y_{\mathrm{C3}}^3\sigma_5 - \mathrm{c}(q_3)\sigma_1 - \\ z_{\mathrm{C3}}^3\sigma_6\sigma_3 + z_{\mathrm{C3}}^3\,\mathrm{c}(q_2)\sigma_7 + y_{\mathrm{C3}}^3\sigma_3\sigma_2 \end{pmatrix} \\
\hline
f_{4\mathrm{x}}^4 - M_3 \begin{pmatrix} y_{\mathrm{C3}}^3\left(\sigma_6{}^2 + \sigma_3{}^2\right) + \mathrm{c}(q_3)\sigma_8 + x_{\mathrm{C3}}^3\sigma_5 + z_{\mathrm{C3}}^3\sigma_4 + \\ \mathrm{s}(q_3)\sigma_1 + z_{\mathrm{C3}}^3\sigma_6\sigma_2 + x_{\mathrm{C3}}^3\sigma_3\sigma_2 \end{pmatrix} \\
\hline
f_{4\mathrm{z}}^4 - M_3 \begin{pmatrix} z_{\mathrm{C3}}^3\left(-\sigma_{11} + \dot{q}_1{}^2 + \dot{q}_2{}^2\right) - y_{\mathrm{C3}}^3\sigma_4 - \mathrm{c}(q_2)\sigma_9 + 2\dot{q}_1\dot{q}_2\,\mathrm{c}(q_2) + \\ +l_2\,\mathrm{s}(q_2)\left(\dot{q}_2{}^2\,\mathrm{c}(q_1) + \dot{q}_1{}^2\,\mathrm{c}(q_2)^2 + \dot{q}_1{}^2\,\mathrm{c}(q_1)\mathrm{c}(q_2)^2 + \sigma_{10}\right) - \\ -x_{\mathrm{C3}}^3\sigma_6\sigma_3 + y_{\mathrm{C3}}^3\sigma_6\sigma_2 - x_{\mathrm{C3}}^3\,\mathrm{c}(q_2)\sigma_7 \end{pmatrix}
\end{pmatrix} \tag{11}
$$

The moment of the constraint force:

$$
\overline{n}_3^3 = \begin{pmatrix}
n_{4\mathrm{y}}^4 + f_{4\mathrm{x}}^4 l_3 + I_{\mathrm{xy}}^{3\mathrm{s}}\sigma_2 + \sigma_{12}\left(I_{\mathrm{yz}}^{3\mathrm{s}}\sigma_{12} - I_z^{3\mathrm{s}}\sigma_{10}\right) + \sigma_{10}\left(I_y^{3\mathrm{s}}\sigma_{12} - I_{\mathrm{zy}}^{3\mathrm{s}}\sigma_{10}\right) + \\
+\sigma_{11}\left(I_{\mathrm{yx}}^{3\mathrm{s}}\sigma_{12} + I_{\mathrm{zx}}^{3\mathrm{s}}\sigma_{10}\right) + I_{\mathrm{xz}}^{3\mathrm{s}}\sigma_1 + I_x^{3\mathrm{s}}\sigma_6 - M_3 y_{\mathrm{C3}}^3\sigma_5 + M_3 z_{\mathrm{C3}}^3\sigma \\
\hline
f_{4\mathrm{y}}^4 l_3 - n_{4\mathrm{x}}^4 - I_y^{3\mathrm{s}}\sigma_2 - \sigma_{12}\left(I_{\mathrm{xz}}^{3\mathrm{s}}\sigma_{12} + I_z^{3\mathrm{s}}\sigma_{11}\right) + \sigma_{11}\left(I_x^{3\mathrm{s}}\sigma_{12} + I_{\mathrm{zx}}^{3\mathrm{s}}\sigma_{11}\right) + \\
+\sigma_{10}\left(I_{\mathrm{xy}}^{3\mathrm{s}}\sigma_{12} - I_{\mathrm{zy}}^{3\mathrm{s}}\sigma_{11}\right) + I_{\mathrm{yz}}^{3\mathrm{s}}\sigma_1 - I_{\mathrm{yx}}^{3\mathrm{s}}\sigma_6 + M_3 x_{\mathrm{C3}}^3\sigma_5 - M_3 z_{\mathrm{C3}}^3\sigma_4 \\
\hline
n_{4\mathrm{z}}^4 + I_{\mathrm{zy}}^{3\mathrm{s}}\sigma_2 - \sigma_{12}\left(I_{\mathrm{xz}}^{3\mathrm{s}}\sigma_{10} + I_{\mathrm{yz}}^{3\mathrm{s}}\sigma_{11}\right) + \sigma_{11}\left(I_x^{3\mathrm{s}}\sigma_{10} - I_{\mathrm{yx}}^{3\mathrm{s}}\sigma_{11}\right) + \\
+\sigma_{10}\left(I_{\mathrm{xy}}^{3\mathrm{s}}\sigma_{10} - I_y^{3\mathrm{s}}\sigma_{11}\right) - I_z^{3\mathrm{s}}\sigma_1 - I_{\mathrm{zx}}^{3\mathrm{s}}\sigma_6 + M_3 y_{\mathrm{C3}}^3\sigma_4 - M_3 x_{\mathrm{C3}}^3\sigma_3
\end{pmatrix} \tag{12}
$$

The generalized driving forces for 3R structure:

$$
Q_m^1 = \begin{pmatrix}
I_y^{1\mathrm{s}}\ddot{q}_1 + I_z^{2\mathrm{s}}\ddot{q}_1 + I_z^{3\mathrm{s}}\ddot{q}_1 - n_{4\mathrm{z}}^4\,\mathrm{s}(q_2) - f_{4\mathrm{z}}^4 l_1\,\mathrm{c}(q_2) + \\
+I_x^{3\mathrm{s}}\ddot{q}_1\,\mathrm{c}(q_2)^2 + I_y^{2\mathrm{s}}\ddot{q}_1\cos(q_2)^2 - I_z^{2\mathrm{s}}\ddot{q}_1\,\mathrm{c}(q_2)^2 - \\
-I_z^{3\mathrm{s}}\ddot{q}_1\,\mathrm{c}(q_2)^2 + I_{\mathrm{zx}}^{2\mathrm{s}}\dot{q}_2{}^2\,\mathrm{c}(q_2) + \dfrac{I_{\mathrm{yz}}^{2\mathrm{s}}\ddot{q}_1\,\mathrm{s}(2q_2)}{2} + \\
+\dfrac{I_{\mathrm{zy}}^{2\mathrm{s}}\ddot{q}_1\,\mathrm{s}(2q_2)}{2} - I_{\mathrm{xy}}^{3\mathrm{s}}\dot{q}_2{}^2\,\mathrm{s}(q_2) + I_{\mathrm{yx}}^{2\mathrm{s}}\dot{q}_2{}^2\,\mathrm{s}(q_2) - \\
-n_{4\mathrm{x}}^4\,\mathrm{c}(q_2)\mathrm{c}(q_3) + n_{4\mathrm{y}}^4\,\mathrm{c}(q_2)\mathrm{s}(q_3)
\end{pmatrix} \tag{13}
$$

$$
Q_m^2 = \begin{pmatrix}
f_{4\mathrm{z}}^4 l_2 + I_{\mathrm{yz}}^{2\mathrm{s}}\dot{q}_1{}^2 + n_{4\mathrm{y}}^4\cos(q_3) + n_{4\mathrm{x}}^4\sin(q_3) + \\
+f_{4\mathrm{x}}^4 l_3\cos(q_3) - f_{4\mathrm{y}}^4 l_3\sin(q_3) + I_{\mathrm{yz}}^{3\mathrm{s}}\dot{q}_1{}^2\cos(q_3) + I_{\mathrm{yz}}^{3\mathrm{s}}\dot{q}_3{}^2\cos(q_3) + \\
+I_{\mathrm{xz}}^{3\mathrm{s}}\dot{q}_1{}^2\sin(q_3) + I_{\mathrm{xz}}^{3\mathrm{s}}\dot{q}_3{}^2\sin(q_3) - I_{\mathrm{xy}}^{3\mathrm{s}}\dot{q}_2\dot{q}_3 - I_{\mathrm{yz}}^{2\mathrm{s}}\dot{q}_1{}^2\cos(q_2)^2 - \\
-I_{\mathrm{zy}}^{2\mathrm{s}}\dot{q}_1{}^2\cos(q_2)^2 + \dfrac{I_x^{3\mathrm{s}}\dot{q}_1{}^2\sin(2q_2)}{2} + \dfrac{I_y^{2\mathrm{s}}\dot{q}_1{}^2\sin(2q_2)}{2}
\end{pmatrix} \tag{14}
$$

2.CONCLUSIONS

A significant part of modeling and simulating a robot is the generation of the geometric model, kinematic and dynamic of its mechanical structure. The software module presented in this paper allows modeling the generalized mechanical structure of the robot and automatic generation of the equations of the dynamic model of the robot considered, expressing the generalized driving forces in the robot's torques.

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