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DYNAMIC LAPLACE MODEL OF A REGENERATIVE HEAT EXCHANGER

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Abstract: *The paper presents the dynamic model using multidimensional Laplace transforms that are applied on time and length coordinate. This approach is very useful to analyze the dynamics, stability of the model of REHEX using linear equations in Laplace domain. More important is the fact that the system no longer needs to be re-integrated if the input variables are changing. The solution is presented for counterflow regenerative heat exchanger.*

Keywords: *dynamic system, Laplace transform, heat exchanger*

1. INTRODUCTION

The regenerative heat exchanger (REHEX) is a heat transfer device that is used in all refrigerating type installations with mechanical vapor compression. It is used both in the field of industrial cold, in the field of commercial and domestic cold. The REHEX is part of the category of heat transfer devices with no phase change and aims to produce total or partial heat regeneration of the vapors of a refrigerant.

Refrigerant vapors, which enter the REHEX, are in most cases superheated. Due to the heat given to the cooling fluid, these vapors cool down to the saturation state corresponding to the condensation temperature. Subsequently, the condensation of the saturated vapors takes place and, finally, a subcooling of the condensed liquid after traversing the REHEX. All these transformations, which take place for the refrigerant vapor part, determine for this type of device the consideration of 3 distinct zones, but in strong heat transfer interaction in between.

2. MATERIALS AND METHODS

The model of the regenerative heat exchanger (REHEX) with countercurrent circulation is considered as a dynamic system with distributed parameters (Fig.1). That approach involves system description with the help of conservation equations, mass and energy, considering in this case both the temporal variable and the spatial variable. The spatial variable for this type of device is considered the length in the dominant flow direction of the cooled and heated fluid.

The modeling assumptions that were used for the REHEX model are: the device is double pipe type, the liquid refrigerant flows through the inner pipe and the refrigerant vapors flows through the space between the inner pipe and the jacket, refrigerant vapors and liquid are considered saturated, the jacket is considered perfectly thermally insulated, heat transfer is considered predominantly radial, the axial heat transfer is neglected, vaporization does not occur in the pipes with liquid refrigerant.

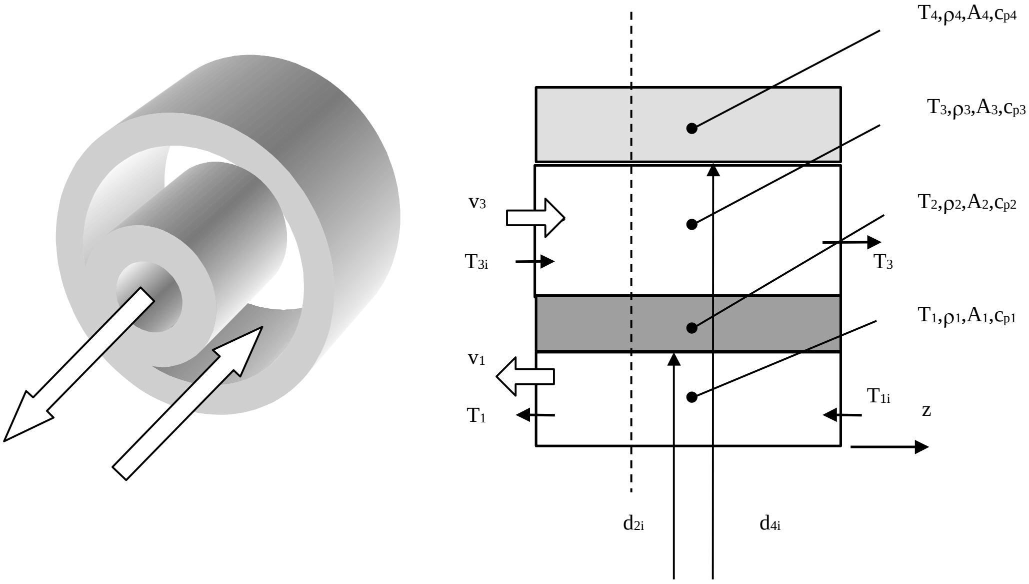

Figure 1: The element of length dℓ for the counterflow REHEX

For the first step of modeling, we consider the energy conservation equation in differential form for an element of infinitesimal length dℓ:

$$-M_L c_{pL} \frac{\partial \theta_L}{\partial t} dl + l_1 \dot{m}_L c_{pL} \frac{\partial \theta_L}{\partial l} dl = \alpha_L A_L (\theta_L - \theta_{per}) dl$$

$$-M_{per} c_{Pper} \frac{\partial \theta_{per}}{\partial t} dl = \alpha_L A_L (\theta_L - \theta_{per}) dl - \alpha_v A_v (\theta_{per} - \theta_v) dl \qquad (1)$$

$$-M_v c_{Pv} \frac{\partial \theta_v}{\partial t} dl + l_2 \dot{m}_v c_{Pv} \frac{\partial \theta_v}{\partial l} dl = \alpha_v A_v (\theta_{per} - \theta_v) dl$$

where M represents mass [kg], $\dot{m}$ is mass flow [kg s^{-1}], c_p is the specific heat [kJ kg^{-1} K^{-1}], A is the heat transfer area [m^2], ℓ is the current length of the fluid [m], α is the convection heat transfer coefficient [W m^{-2} K^{-1}], θ is the temperature [K]. The subscripts for variables in the above equations refers to liquid (L), vapors (V) and pipe wall (Per).

Since the heat transfer takes place between the refrigerant in the liquid phase (L) and that in the vapor phase (V), without a phase change, it can be considered, with a fairly good approximation, that the variation of the convection coefficients is influenced only by the fluid mass flow. Thus, this dependence can be presented in the form:

$$\alpha_L = \alpha_{L0} + \left(\frac{\partial \alpha_L}{\partial \dot{m}_L} \right)_0 \Delta \dot{m}_L$$

$$\alpha_v = \alpha_{v0} + \left(\frac{\partial \alpha_v}{\partial \dot{m}_v} \right)_0 \Delta \dot{m}_v \qquad (2)$$

where α_{L0}, α_{v0} are the initial values of the convection coefficients.

The linearization of the expression (2) is possible because small variations of the mass flow rate around a nominal value are considered in the dynamic model simulation, and it is allowed to neglect the higher order terms obtained by the Taylor series development of the convection coefficient depending on the variation of mass flow rate.

Next step of modeling is the linearization of the equations (1) around the nominal operating regime, that allows obtaining a form of the system of equations depending on the variations of the state and input quantities. The problem that occurs is that two independent variables appear, time and coordinate, but also the partial derivatives in relation to them. The resolution of this "impediment" can be solved analytically by using integral transformations like Laplace transform.

More precisely, the multidimensional Laplace transform is used. If we note in the Laplace domain, the image [1,3] of the time variable t with p, and the image of the coordinate variable ℓ with s.

2.1. The Laplace transform according to the time coordinate

Applying first, the Laplace transform according to the time coordinate to the system (1) results:

$$-(T_1 p + 1)\Delta\theta_L(L, p) + a_1 \frac{\partial \Delta\theta_L(L, p)}{\partial L} = -\Delta\theta_{per}(0, p) + a_2 \Delta\dot{m}_L(0, p)$$

$$(T_2 p + 1)\Delta\theta_v(L, p) + b_1 \frac{\partial \Delta\theta_v(L, p)}{\partial L} = \Delta\theta_{per}(0, p) + b_2 \Delta\dot{m}_v(0, p) \qquad (3)$$

$$\Delta\theta_{per}(0, p) = \frac{1}{T_{per} p + c_1} \Delta\theta_L(L, p) + \frac{c_2}{T_{per} p + c_1} \Delta\dot{m}_L(0, p) +$$

$$+ \frac{c_3}{T_{per} p + c_1} \Delta\theta_v(L, p) - \frac{c_4}{T_{per} p + c_1} \Delta\dot{m}_v(0, p)$$

where by T_i are the coefficients of the derivative with respect to time of the state variables were noted, and a_i, b_i, c_i are the coefficients that were obtained by normalizing the system (1):

$T_1 = \frac{(m_L c_{p,L})}{(\alpha_{L,0} A_L)}$, $T_{per} = \frac{(m_{per} c_{p,per})}{(\alpha_{L,0} A_L)}$, $T_2 = \frac{(m_V c_{p,V})}{(\alpha_{V,0} A_V)}$, $a_1 = \frac{(\dot{m}_L c_{p,L})}{(\alpha_{L,0} A_L)}$, $b_1 = \frac{(\dot{m}_V c_{p,V})}{(\alpha_{V,0} A_V)}$, $c_3 = \frac{(\alpha_{V,0} A_V)}{(\alpha_{L,0} A_L)}$, $c_1 = 1 + c_3$, $a_2 = \frac{1}{(\alpha_{L,0} A_L)} [A_L (\theta_L - \theta_{per})_0 \left(\frac{\partial \alpha_L}{\partial \dot{m}_L}\right)_0 - c_{pL} \frac{\partial \theta_{L0}}{\partial L}]$,

$b_2 = \frac{1}{(\alpha_{V,0} A_V)} [A_V (\theta_{per} - \theta_V)_0 \left(\frac{\partial \alpha_V}{\partial \dot{m}_V}\right)_0 - c_{pV} \frac{\partial \theta_{V0}}{\partial L}]$, $c_2 = \frac{1}{\alpha_{L0}} [(\theta_L - \theta_{per})_0 \left(\frac{\partial \alpha_L}{\partial \dot{m}_L}\right)_0]$, $c_4 = \frac{1}{(\alpha_{L,0} A_L)} [A_V (\theta_{per} - \theta_V)_0 \left(\frac{\partial \alpha_V}{\partial \dot{m}_V}\right)_0]$, (4)

Thus, the transition to the Laplace domain of the system of differential equations was achieved, the system becomes linear in the variable p (image of the time variable).

2.2. The Laplace transform according to the length coordinate

Applying second, the Laplace transform according to the length coordinate to the system (3) results:

$$[a_1 s - A(p)]\Delta\theta_L(s,p) = -a_1(p)\Delta\theta_v(s,p) + \frac{a_2(p)}{s}\Delta\dot{m}_v(0,p) + \\ + \frac{a_3(p)}{s}\Delta\dot{m}_L(0,p) + \Delta\theta_{L,in}(0,p)$$

$$[b_1 s + B(p)]\Delta\theta_v(s,p) = b_4(p)\Delta\theta_L(s,p) + \frac{b_5(p)}{s}\Delta\dot{m}_L(0,p) + \\ + \frac{b_6(p)}{s}\Delta\dot{m}_v(0,p) + \Delta\theta_{v,in}(1,p) \quad (5)$$

Finally, is obtained a dependence of the variation of the state variables according to s and p Laplace variables.

The system (5), obtained in the Laplace domain, is a system of linear equations where the state variables, the temperature variation of the liquid agent and vapors, depend on the inlet temperatures and agent flow rates. Solving this system of linear equations is done by successive substitutions. Thus, it obtains the variation of the state quantities depending on the input quantities in the domain (s,p). Obtaining the solution in the initial domain time-length (t,x) is achieved by applying the inverse two-dimensional Laplace transform [3].

The analytical solution for $\Delta\theta_L(t,x)$ and $\Delta\theta_V(t,x)$ obtained by the inverse Laplace transform is quite complicated, but more important fact is it describes the temperature field for the REHEX in dynamic mode and depending on the dominant coordinate.

3. TRANSFER FUNCTION MODEL

It is more convenient to describe this solution in the form of transfer functions. By definition21, the transfer function represents the ratio between the Laplace transform of the output quantity and the Laplace transform of the input quantity. The representation of the solution of the system of differential equations (5) in the form of transfer functions for the regenerative heat exchanger with countercurrent flow is:

$$\Delta\theta_V(s,p) = AA_1\,\Delta\dot{m}_v(0,p) + AA_2\Delta\dot{m}_L(0,p) + AA_3\Delta\theta_{L,in}(0,p) + AA_4\,\Delta\theta_{v,in}(1,p)$$

$$\Delta\theta_L(s,p) = BB_1\,\Delta\dot{m}_v(0,p) + BB_2\Delta\dot{m}_L(0,p) + BB_3\Delta\theta_{L,in}(0,p) + BB_4\,\Delta\theta_{v,in}(1,p) \qquad (6)$$

where: AAi, BBi, are the transfer functions of the output quantities, in our case the variation of the vapor outlet temperature $\Delta\theta_V$ and the variation of the liquid outlet temperature $\Delta\theta_L$, in relation to the input quantities: the variation of the liquid mass flow rate $\Delta\dot{m}_L$, the variation of the vapor mass flow rate $\Delta\dot{m}_V$, the variation of the temperature of the liquid at the inlet $\Delta\theta_{L,in}$, and the variation of the temperature of the vapors at the inlet of the exchanger $\Delta\theta_{V,in}$.

The AAi, BBi coefficients have the following expressions:

$$AA_1 = \frac{(aa_2B(p)-aa_1bb_6)}{s_1s_2} + \frac{1}{(s_2-s_1)}\{[\frac{(aa_2B(p)-aa_1bb_6)}{(-s_1)} + aa_2b_1]\exp(-s_1L) -$$

$$-[\frac{(aa_2B(p)-aa_1b_6)}{(-s_2)} + aa_2b_1]\exp(-s_2L)\};$$

$$AA_2 = \frac{(aa_3B(p)-aa_1bb_5)}{s_1s_2} + \frac{1}{(s_2-s_1)}\{[\frac{(aa_3B(p)-aa_1bb_5)}{(-s_1)} + aa_3b_1]\exp(-s_1L) -$$

$$-[\frac{(aa_3B(p)-aa_1bb_5)}{(-s_2)} + aa_3b_1]\exp(-s_2L)\};$$

$$AA_3 = \frac{1}{(s_2-s_1)}[(-s_1b_1+B(p))\exp(-s_1L) - (-s_2b_1+B(p))\exp(-s_2L)];$$

$$AA_4 = \frac{aa_1}{(s_2-s_1)}[\exp(-s_1L) - \exp(-s_2L)]; \qquad (7)$$

$$BB_1 = \frac{aa_2bb_4-A(p)bb_6}{s_1s_2} + \frac{1}{(s_2-s_1)}\{[\frac{(aa_2bb_4-A(p)bb_6)}{(-s_1)} + a_1bb_6]\exp(-s_1L) -$$

$$-[\frac{(aa_2bb_4-A(p)bb_6)}{(-s_2)} + a_1bb_6]\exp(-s_2L)\};$$

$$BB_2 = \frac{aa_3bb_4-A(p)bb_5}{s_1s_2} + \frac{1}{(s_2-s_1)}\{[\frac{(aa_3bb_4-A(p)bb_5)}{(-s_1)} + aa_1bb_5\exp(-s_1L) -$$

$$-[\frac{(aa_3bb_4-A(p)bb_5)}{(-s_2)} + aa_1bb_5]\exp(-s_2L)\};$$

$$BB_3 = \frac{bb_4}{(s_2-s_1)}[\exp(-s_1L) - \exp(-s_2L)];$$

$$BB_4 = \frac{1}{(s_2-s_1)}[(-s_1a_1-A(p))\exp(-s_1L) - (-s_2a_1-A(p))\exp(-s_2L)];$$

and are expressed as transfer functions only as a function of the complex variable p, since the inverse Laplace transform after the variable s has already been applied.

The quantities s_1, s_2 represent the solutions of the equation obtained if we consider as the determined characteristic of the system of equations (6) to be zero. These solutions of the equation (called characteristic equation) are called the poles of the dynamic system in Laplace coordinates. These values are very important for the analysis of the system, because depending on their value, an analysis is made regarding the stability of the dynamic system [2-7].

The value of the poles of dynamic system is:

$$s_{1,2} = -\frac{a_1B(p)-b_1A(p)}{2a_1b_1} \pm\sqrt{\Delta} \qquad (8)$$

where $\Delta =\{[\frac{a_1B(p)-b_1A(p)}{2a_1b_1}]^2+[\frac{A(p)B(p)-aa_1bb_4}{a_1b_1}]\}$

For example, the coefficient AA1 in the system (7) represents the influence of the mass flow rate variation of vapors, $\Delta\dot{m}_v$, on the state quantity $\Delta\theta_v$ (variation of the refrigerant vapor temperature), considered in the Laplace domain. Similarly, the other coefficients represent the influences introduced by the corresponding disturbing quantities on the state quantities.

4. CONCLUSIONS

The paper presents the dynamic model using multidimensional Laplace transforms that are applied on time and length coordinate. This approach is very useful to analyze the dynamics, stability of the model of REHEX using linear equations in Laplace domain. More important is the fact that the system no longer needs to be re-integrated if the input variables are changing.

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