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MATRIX STRAIN INCREASE FACTOR OF COMPOSITE LAMINAE

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Abstract: *Matrix strain increase factors (MSIFs) for two types of fibers disposal in composite laminae subjected to transverse tensile loads have been computed. A computing method of hexagonal shape disposed fibers in unidirectional (UD) reinforced lamina has been developed. A comparison between the MSIF in case of square shape and hexagonal shape disposed fibers in UD glass/epoxy and HM-carbon/epoxy laminae is accomplished. A significantly difference between MSIFs at higher fibers volume fractions can be noted.*

Keywords: *lamina, transverse load, glass fibers, HM-carbon fibers*

1. INTRODUCTION

The basic element of a laminate composite structure is represented by an individual layer called lamina reinforced with continuous unidirectional fibers inserted into a resin system called matrix [1-4].

In a general case, the plane loading of a lamina is formed by three components (longitudinal, transverse and shear) that have been connected to the main directions in material. From a macroscopic point of view, a lamina can be imagined as a continuous anisotropic homogeneous element in which the infinitely long fibers are inserted into matrix [5-9].

On fibers subjected to transverse tensile loading act different influences affecting the lamina's mechanical behavior, namely stresses due to the loading action, interference stresses, possible internal stresses as well as additional stresses that cause an increase in the matrix strain. Since an element from the composite laminate having parallel disposed fibers is deformed under a transverse tensile loading, the external measurable deformations should also take place within material. The matrix strain increase factor (MSIF) represents a ratio between the matrix transverse strain and the overall strain perpendicular to the fibers' direction [10-13].

2. MATRIX STRAIN INCREASE FACTOR’S COMPUTING METHOD

To compute the MSIF in case of a parallel hexagonal shape disposed fibers in a unidirectional (UD) fibers-reinforced lamina, a scheme presented in Fig.1 has been developed. The maximum fibers volume fraction in this case can be computed using the scheme shown in Fig. 2.

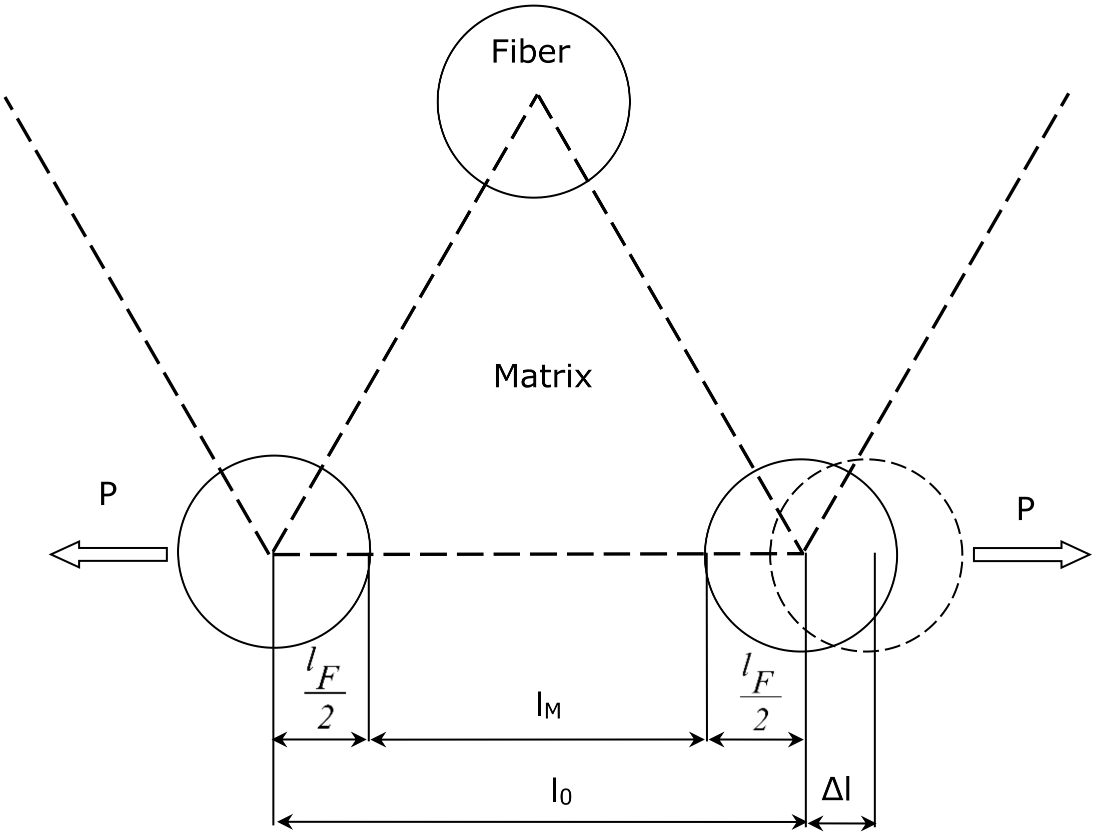

Figure 1: Parallel hexagonal shape disposed fibers in a lamina subjected to transverse loads

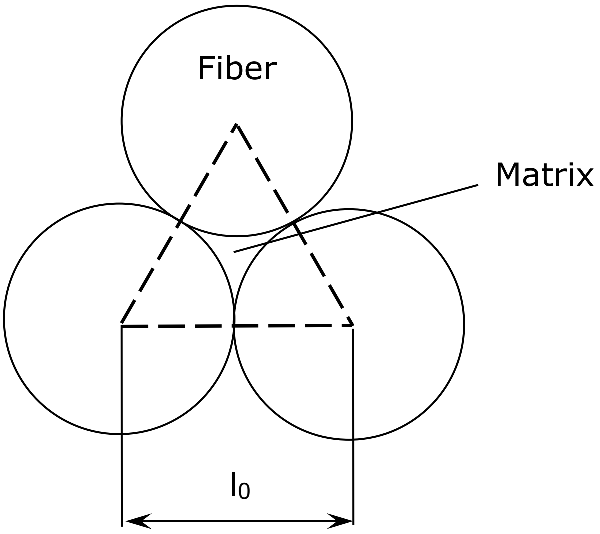

Figure 2: Maximum fibers volume fraction in case of parallel hexagonal shape disposed fibers

We can start from the fact that:

$$\frac{\pi d^2}{8} = \varphi \cdot \frac{l_0^2 \cdot \sqrt{3}}{4}, \tag{1}$$

where φ represents the fibers volume fraction.

So, the maximum fibers volume fraction in case of parallel hexagonal shape disposed fibers in a UD fibers-reinforced lamina subjected to transverse tensile loads will be

$$\varphi =\cdot \frac{\pi d^2}{2l_0^2 \cdot \sqrt{3}}. \tag{2}$$

From equation (2), following important ratio can be determined:

$$\frac{d}{l_0} = \sqrt{\frac{2\varphi\sqrt{3}}{\pi}}. \tag{3}$$

Relation (3) gives the theoretical maximum fibers volume fraction in case of hexagonal shape disposed fibers in a UD fibers-reinforced lamina in which the ratio $d/l_0 = 1$:

$$\varphi_{max} = \frac{\pi}{2\sqrt{3}} = 0.906 \tag{4}$$

The ratio between the lengths of matrix and fibers can be computed using the relation below:

$$\frac{l_M}{l_0} = 1 - \frac{d}{l_0} = 1 - \sqrt{\frac{2\varphi\sqrt{3}}{\pi}}. \tag{5}$$

So, in this case, the MSIF of isotropic fibers, will be:

$$f_\varepsilon = \frac{\varepsilon_M}{\varepsilon_\perp} = \frac{1}{\frac{l_M}{l_0} + \frac{E_M}{E_F}\left(1 - \frac{l_M}{l_0}\right)} = \frac{1}{\frac{l_M}{l_0}\left(1 - \frac{E_M}{E_F}\right) + \frac{E_M}{E_F}} = \frac{1}{\left(1 - \sqrt{\frac{2\varphi\sqrt{3}}{\pi}}\right)\left(1 - \frac{E_M}{E_F}\right) + \frac{E_M}{E_F}}. \tag{6}$$

For instance, in case of HM-carbon fibers-reinforced epoxy resin lamina, due to strong anisotropy of these fibers, the MSIF for both hexagonal and square shape disposed fibers, will be:

$$f_{\varepsilon\ hexagonal} = \frac{\varepsilon_M}{\varepsilon_\perp} = \frac{1}{\frac{l_M}{l_0} + \frac{E_M}{E_{F\perp}}\left(1 - \frac{l_M}{l_0}\right)} = \frac{1}{\frac{l_M}{l_0}\left(1 - \frac{E_M}{E_{F\perp}}\right) + \frac{E_M}{E_{F\perp}}} = \frac{1}{\left(1 - \sqrt{\frac{2\varphi\sqrt{3}}{\pi}}\right)\left(1 - \frac{E_M}{E_{F\perp}}\right) + \frac{E_M}{E_{F\perp}}}. \tag{7}$$

$$f_{\varepsilon\ square} = \frac{\varepsilon_M}{\varepsilon_\perp} = \frac{1}{\frac{l_M}{l_0} + \frac{E_M}{E_{F\perp}}\left(1 - \frac{l_M}{l_0}\right)} = \frac{1}{\frac{l_M}{l_0}\left(1 - \frac{E_M}{E_{F\perp}}\right) + \frac{E_M}{E_{F\perp}}} = \frac{1}{\left(1 - 2\sqrt{\frac{\varphi}{\pi}}\right)\left(1 - \frac{E_M}{E_{F\perp}}\right) + \frac{E_M}{E_{F\perp}}}. \tag{8}$$

3. RESULTS

A comparison between the MSIF in case of square shape and hexagonal shape disposed fibers for a UD glass fibers-reinforced epoxy resin lamina subjected to transverse tensile loads is presented in Table 1 and shown in Fig. 3. Following input data for Young's moduli are: E_M = 3 GPa; E_F = 73 GPa.

Table 1. MSIF for two shapes of disposed fibers in a UD glass/epoxy lamina

φ (%)	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**
$f_{\varepsilon\ \mathrm{hexagon}}$	1	1.46	1.81	2.22	2.75	3.47	4.54	6.34
$f_{\varepsilon\ \mathrm{square}}$	1	1.52	1.93	2.45	3.16	4.25	6.17	10.55

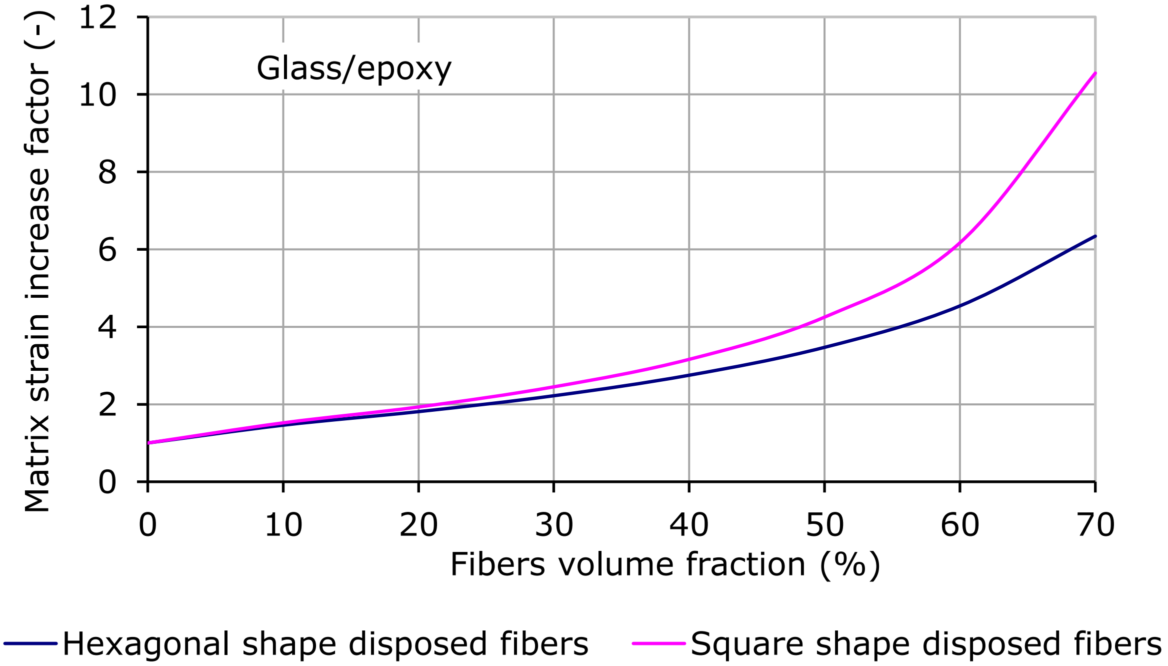

Figure 3: MSIF for hexagonal and square shape disposed fibers in a glass/epoxy lamina

For HM-carbon fibers-reinforced epoxy resin lamina, following input data have been used to compute the MSIF for both hexagonal and square shape disposed fibers: E_M = 3 GPa; $E_{F\perp}$= 25 GPa.

A comparison between the MSIF in case of square shape and hexagonal shape disposed fibers for a UD-HM carbon fibers-reinforced epoxy resin lamina subjected to transverse tensile loads is presented in Table 2 and visualized in Fig. 4. The comparison of both MSIF in case of both types of fibers disposal are presented in Figs. 5-6.

Table 2. MSIF for two shapes of disposed fibers in a UD HM-carbon/epoxy lamina

φ (%)	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**
$f_{\varepsilon\ \mathrm{hexagon}}$	1	1.41	1.7	2.02	2.4	2.88	3.51	4.4
$f_{\varepsilon\ \mathrm{square}}$	1	1.45	1.79	2.19	2.68	3.35	4.33	5.9

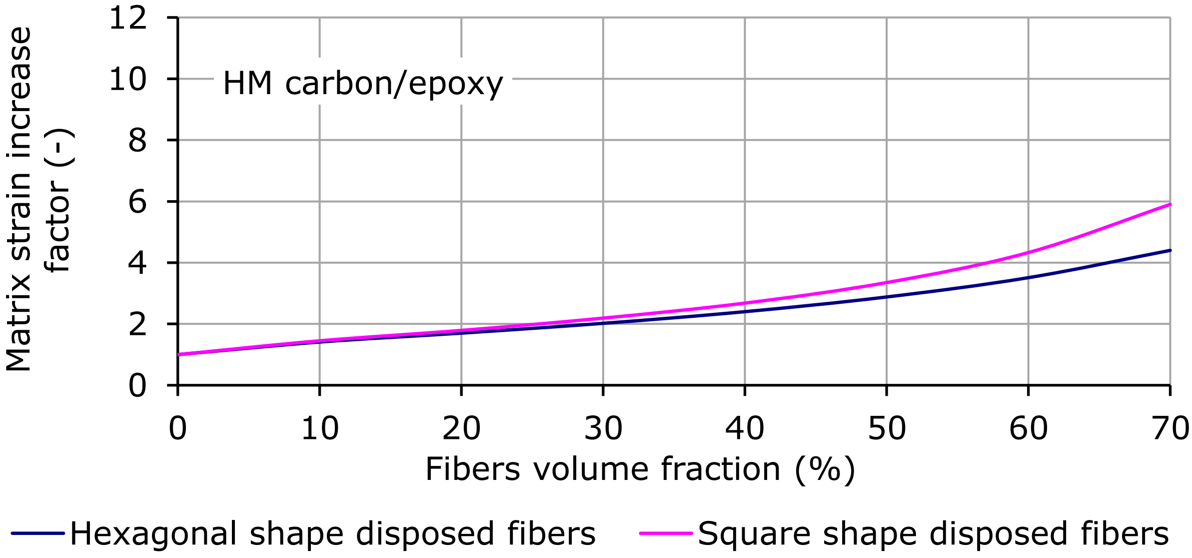

Figure 4: MSIF for hexagonal and square shape disposed fibers in a HM-carbon/epoxy lamina

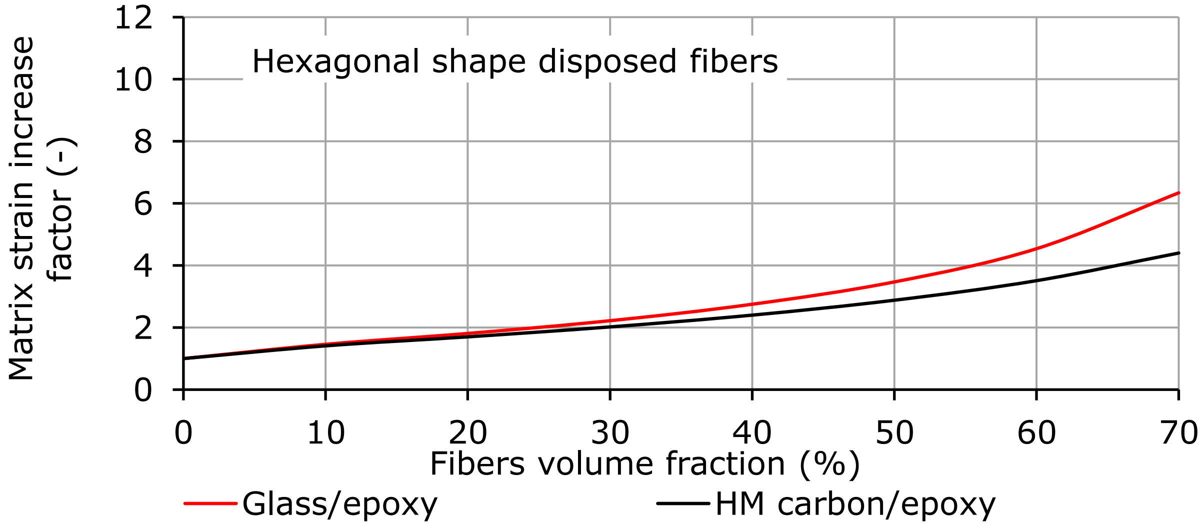

Figure 5: MSIF in HM-carbon and glass/epoxy laminae for hexagonal shape disposed fibers

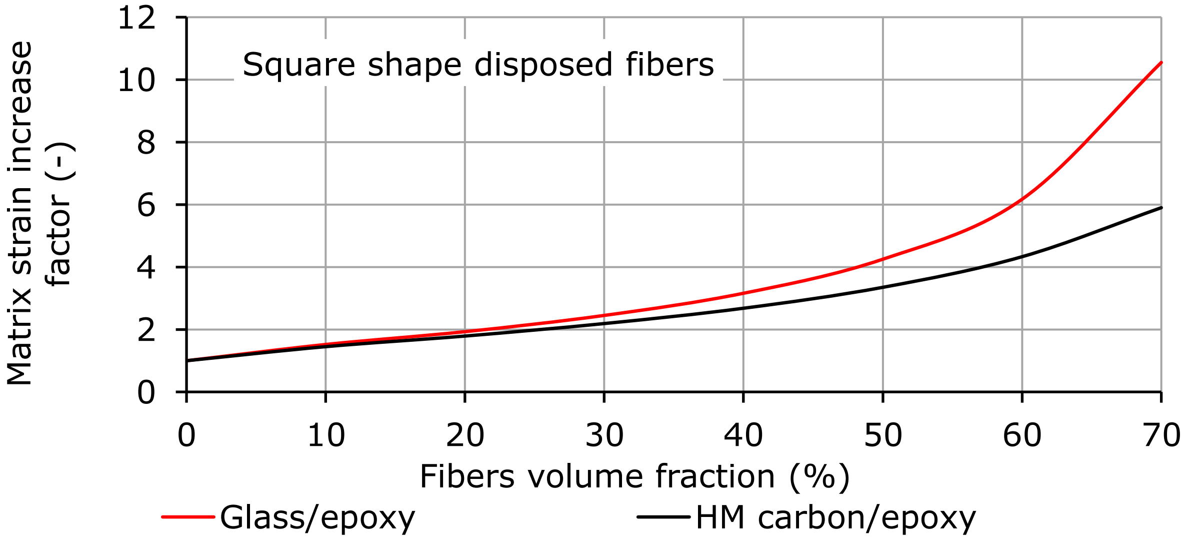

Figure 6: MSIF in HM-carbon and glass/epoxy laminae for square shape disposed fibers

4. CONCLUSIONS

With the increase of fibers volume fraction in a composite lamina, the MSIF increases also. In case of hexagonal shape disposed fibers, the MSIF presents lower values than the square shape disposed ones, which indicates that the hexagonal shape is more advantageous being closer to reality. Moreover, at fibers volume fractions over 40%, the difference between these two MSIFs is considerable. This fact is more significant in case of glass/epoxy than the HM-carbon/epoxy, as well as in case of square shape disposed fibers than the hexagonal shape disposed ones.

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