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ON MAXIMUM POWER FOR AN ELECTROAEOLIAN SYSTEM OPERATING AT VARIABLE WIND SPEED VALUES

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Abstract: This paper proposes a simple method for determining the performance of a system composed of a wind turbine – (WT) and a permanent magnet synchronous generator (PMSG). By using the WT and PMSG characteristics, the system equations are deduced, from which can be calculated the basic variables: speed, currents, voltages, couples, operating at points of maximum power for specific wind speed values. For a certain wind speed, one can actually calculate all variables that define the points of maximum power operation

Key words: WT model; PMSG model, maximal power, estimation of performance, wind speed

1. INTRODUCTION

Eolian systems are becoming more and more frequent in areas with a multianual wind speed over 3 [m/s] [2]. As the maximal power developed by a wind turbine (WT) depends on the cubic value of the wind speed, it is very important that the system formed by the Wind Turbine (WT) and the synchronic generator (WT + SG) function in the maximal power points [3].

Mathematical models

Below are presented the mathematical models for the turbine and generators.

Wind Turbine – WT – Mechanical traits of the WT in the working area are represented as right lines [2].

$$M_T = A\omega + B \tag{1}$$

where: A, B – constant; ω – mechanic angular speed

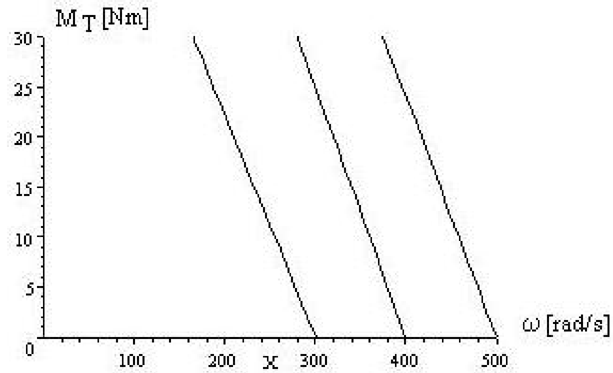

Fig.1: The couple of eolian turbine

The turbine – WT – can develop for a certain wind speed a maximal power of:

$$M_T = A\omega + B \quad P_{TV_{\max}} = M_{TV}^* \cdot \omega^* = -\frac{B^2}{2A} \tag{2}$$

a relation derived from:

$$\frac{dP_{TV}}{d\omega} = \frac{d(A\cdot\omega + B)\cdot\omega}{d\omega} = 0 \Rightarrow \omega^* = -\frac{B}{2A} \tag{3}$$

The synchronic generator with permanent magnets (PMSG):

In a dynamic mode the PMSG is characterized (orthogonal model) by the equations:

$$\begin{cases} -U\sqrt{3}\sin\theta = R_d \cdot I_d + L_d \cdot \dfrac{dI_d}{dt} - \omega \cdot L_q \cdot I_q + M_D \cdot \dfrac{dI_d}{dt} - \omega \cdot M_Q \cdot I_Q \\ U\sqrt{3}\cos\theta = \omega \cdot L_d \cdot I_d + R_q \cdot I_q + L_q \cdot \dfrac{dI_q}{dt} + \omega \cdot M_D \cdot I_D + M_Q \cdot \dfrac{dI_Q}{dt} + \omega \cdot \Psi_{MP} \\ 0 = M_D \cdot \dfrac{dI_d}{dt} + R_D \cdot I_D + L_D \cdot \dfrac{dI_D}{dt} \\ 0 = M_Q \cdot \dfrac{dI_q}{dt} + R_Q \cdot I_Q + L_Q \cdot \dfrac{dI_Q}{dt} \\ J\dfrac{d\omega}{dt} = p_1\left[\left(L_d + L_q\right) \cdot I_d \cdot I_q + I_q \cdot \Psi_{MP} - M_Q \cdot I_d \cdot I_Q + M_D \cdot I_q \cdot I_D\right] - M_{motor} \end{cases} \tag{4}$$

The machine’s parameters are:

- R_D - resistance of the amortization winding on d axis;
- R_Q - Resistance of the amortization winding on q axis;
- L_D - the inductance of the amortization winding of d axis;
- M_{Dd} - mutual inductance between the entrance winding “d” and the D winding;
- L_Q - the inductance of the amortization winding on q axis;
- M_{Qq} - mutual inductance between entrance winding “q” and the Q winding;
- R_1 - the resistance of the entrance winding;
- L_d - the inductance of the entrance winding on d axis;
- L_q - the inductance of the entrance winding on q axis;
- Ψ_{sN} - nominal entrance flux;
- Ψ_{MP} - permanent magnet flux;

In PMSG initial conditions (the starting functioning point) are to be deduced considering the algebraic system defined by the initial couple, M_{motor}:

$$\begin{cases} -U\sqrt{3}\sin\theta = R_1 \cdot I_d - \omega \cdot L_q \cdot I_q \\ U\sqrt{3}\cos\theta = R_1 \cdot I_q + \omega \cdot L_d \cdot I_d + \omega \cdot \Psi_{M} \\ M_{motor} = p_1 \cdot (L_d - L_q) \cdot I_d \cdot I_q + I_q \cdot \Psi_{M} \end{cases} \quad (5)$$

2. MAXIMAL POWER FUNCTIONING

The maximal power that can be developed – WT – depends on the cubic wind speed (V) [9] and the square of the angular rotation speed ω (or Ω), as can be seen below

$$P_{max} = K_p \cdot V^3 \quad (6)$$

The points of maximum P_1, P_2, P_3 are related to the V_1, V_2, V_3 wind speeds and angular rotation speed ω_1^n, ω_2^n, ω_3^n.
The driving strategy for any electrical power generating eolian system implies the functioning of the system in points P_k, corresponding to the maximal power

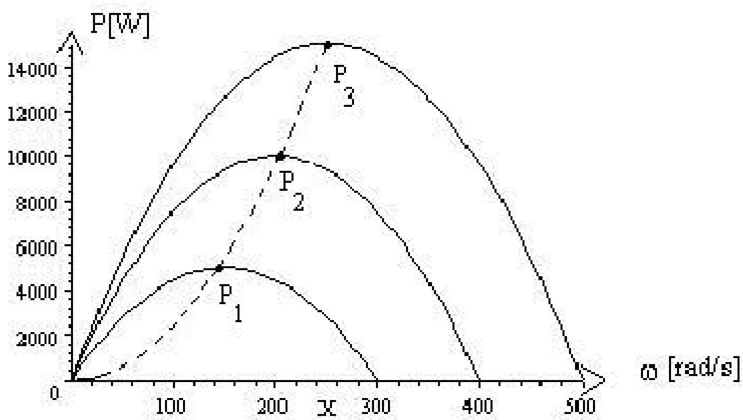

Fig.2: Variation of power in relation with angular speed

A stable mode of functioning for the WT+AG system is reached in the $\omega \geq \omega_1^n$ area.
The load at PMSG can be:

1) An array of electrical batteries – AE;
2) Loading resistance – R;

2.1.The WE + PMSG + AE system

The driving system implies the estimation of optimal rotation $-n^*(\omega^* = 2\pi n^*)$ – at a wind speed – V – and the command of the rectifier located between the PMSG and the electrical batteries – AE, so that the coordinates for the point of maximal power be reached: n^*, M_T^n.

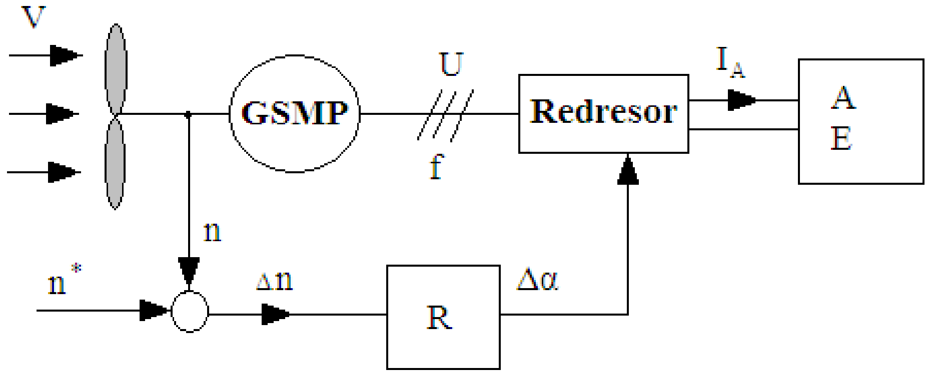

Fig.3 : Driving of system WT + PMSG + AE

2.2. The system composed of WT+PMSG+R, with R = load resistor

In this configuration at the PMSG's terminals a load resistor is attached – the resistor must be variable, in order to function at maximal power at a variable wind speed

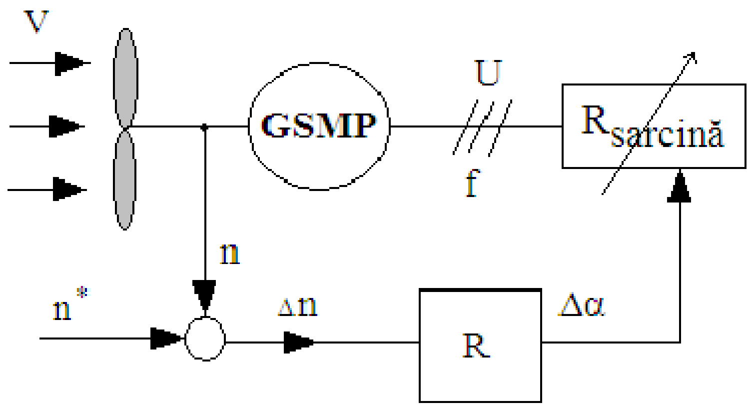

Fig.4 : Driving of system WT + PMSG + R

2.3.Wind speed dependence of the couple.

In the functioning area can be considered a linear mechanical parameter $M_{TV}(\omega)$, for WT of the following type: [2]

$$M_{TV} = a \cdot \omega + K_1 \cdot V^{\alpha} \tag{7}$$

where: - ω - angular mechanic speed of the WT's axis;

- a, K_1, α - coefficients depending on the turbine's geometry; to be calculated according to catalog data (as presented below):

The coordinates for the point of maximal power, M_{TV}^* and ω^* are calculated with:

$$\frac{dP}{dt} = 0 \Rightarrow \omega^* = -\frac{K_1 \cdot V^{\alpha}}{2a}; M_{TV}^* = \frac{K_1 \cdot V^{\alpha}}{2} \tag{8}$$

And therefore $P_{\max}$ is:

$$P_{\max} = M_{TV}^* \cdot \omega^* = -\frac{K_1^2 \cdot V^{2\alpha}}{4a} = K_p \cdot V^3 \tag{9}$$

From which: $\alpha = 1{,}5$ şi deci $M_{TV} = a \cdot \omega + K_1 \cdot V^{1,5}$.

3. CASE STUDY

3.1. Case 1 – The coordinates of the maximal power point:

Catalog data for the turbine [11]:

$\omega_0 = 40$ [rad/s] – angular speed at no-load functioning;

$\omega_{ref} = 30$ [rad/s] – nominal (reference) angular speed for $V_{ref} = 5$ [m/s];

$M_{ref} = 5$ [Nm] – nominal couple for $V = 5$ [m/s].

No-load functioning: $M_{TV} = a \cdot \omega + K_1 \cdot V^{1,5}$ and:

$$0 = a \cdot 40 + K_1 \cdot 5^{1,5} \tag{10}$$

For $M_{ref} = 5$ [Nm], $V_{ref} = 5$ [m/s] it results:

$$5 = a \cdot 30 + K_1 \cdot 5^{1,5} \tag{11}$$

From the equations above it results:

$$a = -0,5, \quad K_1 = \frac{20}{5^{1,5}} \tag{12}$$

Finally, the mechanical parameter of the WT comes from the following equation:

$$M_{TV} = -0,5 \cdot \omega + 20 \cdot \left(\frac{V}{5}\right)^{1,5} \tag{13}$$

At a wind speed – V – the power of the turbine is:

$$P_{TV} = M_{TV} \cdot \omega \tag{14}$$

And it is maximal at: $\omega^* = 20 \cdot \left(\frac{V}{5}\right)^{1,5}$ [rad/s], $M_{TV}^* = 10 \cdot \left(\frac{V}{5}\right)^{1,5}$ (15)

Which derives from $\frac{dP_{TV}}{d\omega} = 0$ (16)

The coordinates of the maximal power point for V = 5 [m/s] are:

$$\omega^* = -\frac{K_1 \cdot V^{1,5}}{2a} = 20 \text{ [rad/s]}, \; M_{TV}^* = -\frac{k_1 \cdot V^{1,5}}{2} = 10 \text{ [Nm]}, \; P_{TV}^* = 49 \cdot 245 \left(\frac{V}{4}\right)^3 \tag{17}$$

PMSG will work at a frequency of: $f = \frac{p_1 \omega^*}{(2\pi)} \cdot K_{red}$,

Where: p_1 - the number of the PMSG's pairs of poles;

$K_{red} = \frac{314}{2\pi}$ - the multiplication coefficient for the regulator located between WT and PMSG.

The PMSG couple must become K_{red} times lower than the couple for the WT:

$$M_{GSMP} = \frac{M_{TV}}{K_{red}} = \frac{bV^{1,5}}{2} / K_{red} = 10\left(\frac{V}{5}\right)^{1,5} / K_{red} \tag{18}$$

Study Case 2 – The functioning of the WT + PMSG system for a wind speed V = 5 [m/s]

Turbine data:

$V_N = 5$ [m/s] – wind speed; $\omega^* = -\frac{k_1 \cdot V^{1,5}}{2a} = 20$ [rad/s] for WT, and for the PMSG (through the regulator) $\omega = 314$ [rad/s];

$M_{TV}^{n} = \frac{k \cdot V^{1,5}}{2} = 10$ [Nm] for WT, and for the PMSG through the regulator it results

$M_{PSMG} = 10 \cdot \frac{20}{314} = 0,6369$ [Nm].

PMSG data:

$L_d = 0,07$ [H] – synchronic reactance in d axis; $L_q = 0,08$ [H] – synchronic reactance in q axis; $\Psi_{MP} = 1,3$ [Wb] – the permanent magnet flux

The PMSG equations are [1]:

$$\begin{cases} U_d = 1,6 I_d - \omega \cdot 0,08 I_q \\ U_q = 1,6 I_q + \omega \cdot 0,07 I_d + \Psi_M \\ M = -0,01 I_d \cdot I_q + \Psi_M I_q \end{cases} \tag{19}$$

For a wind speed V = 10 [m/s] and the wind turbine in maximal power point functioning, the PMSG is analyzed through the equations:

$$\begin{cases} V = 10 \\ \omega_T = 20\left(\frac{V}{5}\right)^{1,5} \\ U_d = 1,6 I_d - \omega \cdot 0,08 I_q \\ U_q = 1,6 I_q + \omega \cdot 0,07 I_d + \omega \Psi_M \\ -\left(-0,5\omega_T + 20\left(\frac{V}{5}\right)^{1,5}\right)\frac{20}{314} = -0,01 I_d I_q + \Psi_M I_q \\ U_d = -R I_d \\ U_q = -R I_q \\ P = U_d I_d + U_q I_q \\ Q = -U_d I_q + U_q I_d \\ F_P = \frac{P}{\sqrt{P^2 + Q^2}} \\ \omega = 314\left(\frac{V}{5}\right)^{1,5} \\ F = \omega / 6,28 \\ \Psi_M = 1,3 \\ I_R = \frac{\sqrt{I_d^2 + I_q^2}}{\sqrt{3}} \\ U_R = \frac{\sqrt{U_d^2 + U_q^2}}{\sqrt{3}} \\ \Psi_s^2 = (0,08 I_q)^2 + (\Psi_M + 0,07\, I_d)^2 \end{cases} \tag{20}$$

$$\begin{cases} Q = 0;\ F_P = 1,0;\ V = 10;\ \Psi_M = 1,3;\ P = -1596,9;\ \omega = 888,13;\ \omega_T = 56,569; \\ U_q = 1145,0;\ F = 141,42;\ U_R = 663,47;\ U_d = 98,182; R = 826,97; \\ I_q = -1,3845;\ I_R = 0,80230;\ I_d = -0,11873;\ \Psi_s = 1,2964 \end{cases} \tag{21}$$

For a wind speed V = 10 [m/s] and the wind turbine functioning in the maximal power point, the tri-phased PMSG will deliver in the batteries AE an active power – P, a intensity I_R, at the U_R voltage:

V = 10 [m/s]; P = 1596,9 [W]; I_R = 0,8 [A]; U_R = 663,47 [V]; F = 141,42 [Hz].

For a wind speed V = 5 [m/s] and the turbine functioning in the maximal power point the PMSG will deliver an active power in the batteries -AE,:

P = 199,62 [W],

At an intensity, voltage and tri-phased frequency with the following values:

I_R = 0,28297 [A]; U_R = 235,14 [V]; F = 50,00 [Hz].

For a wind speed V = 25 [m/s] and the turbine functioning in the maximal power point the PMSG will deliver an active power in the batteries -AE,:

P = 24947 [W],

At an intensity, voltage and tri-phased frequency with the following values:

I_R = 3,3 [A]; U_R = 2502,20 [V]; F = 559,00 [Hz].

3. MECHANICAL CHARACTERISTICS – EXPERIMENTAL CHECK-UPS

The mechanical characteristic of the WT has the following expression:

$$M_{TV} = a \cdot \omega + b \cdot V^{1,5} \tag{23}$$

Two mechanical characteristics will be checked for wind speed = 4 [m/s] and 5 [m/s], as given below.

For a wind speed V = 4 [m/s] the mechanical characteristic is represented by the line **BC** (B(2,21), C(7,0)):

$$M_{TV-4} = -4{,}2 \cdot \omega + 29{,}4 \tag{24}$$

And for a wind speed V = 5 [m/s] is represented by the line **AD** (A(2,32), D(9,0)):

$$M_{TV-5} = -4{,}5714 \cdot \omega + 41{,}143 \tag{25}$$

For V = 4 [m/s] the maximal power point – P_1 has a value of:

P_1 = 51,45 [W] la $\omega_1^* = 3{,}5$ [rad/s],

And for V = 5 [m/s] the maximal power point – P_2 has a value of:

P_2 = 92,573 [W] la $\omega_2^* = 4{,}5$ [rad/s].

With the two values (P_1 and P_2) the relation between the maximal power and the wind speed, namely:

$$P = k \cdot V^3 \tag{26}$$

Or

$$\frac{P_2}{P_1} = \frac{k5^3}{k4^3} = \left(\frac{5}{4}\right)^3 = 1{,}9 \tag{27}$$

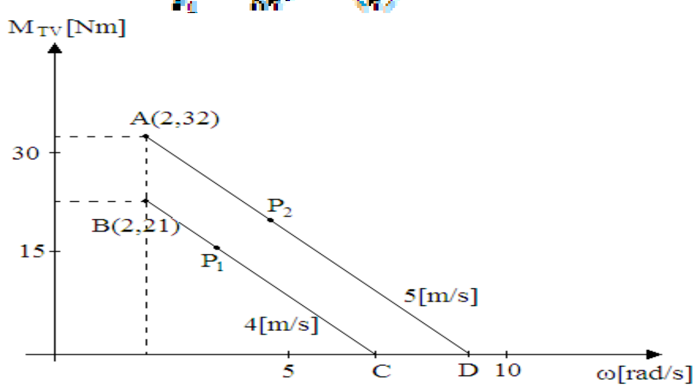

Fig.5 : WT's characteristics for V = 4 and V = 5 [m/s]

From the maximal power point P_1 = 51,45 [W] and P_2 = 92,573 [W] it results:

$$M_{TV-4} = -4{,}2 \cdot \omega + b \cdot V^{\alpha} = -4{,}2\omega + b \cdot 4^{\alpha} = -4{,}2 \cdot \omega + 29{,}4 \tag{28}$$

And

$$M_{TV-5} = -4{,}57 \cdot \omega + b \cdot V^{\alpha} = -4{,}5 \cdot \omega + b \cdot 5^{\alpha} = -4{,}5 \cdot \omega + 41{,}143 \tag{29}$$

From the two relations it results:

$$\begin{cases} 29{,}4 = b \cdot 4^{\alpha} \\ 1{,}143 = b \cdot 5^{\alpha} \end{cases} \tag{30}$$

Or:

$$\frac{29{,}4}{41{,}143} = \left(\frac{4}{5}\right)^{\alpha} \tag{31}$$

From which: $\alpha = 1{,}5$.

As a conclusion, the mechanical characteristic of the WT for an average angle $4,38 = \frac{4,2+4,57}{2}$, can be calculated from the relation:

$$M_{TV} = -4,38 \cdot \omega + 29,4 \cdot \left(\frac{V}{4}\right)^{1,5} \tag{32}$$

The maximal power point for a wind speed - V, derives from the values for :

$\omega^* = 3,35\left(\frac{V}{4}\right)^{1,5}$ – angular speed; $M^*_{TV} = 14,7\left(\frac{V}{4}\right)^{1,5}$ – couple

4. CONCLUSIONS

The paper allows the calculation of performance parameters for a system formed by a Wind Turbine (WT) and a Permanent Magnet Synchronic Generator (PMSG) working at maximal power. The relations for calculating mechanic angular speed (rotation), couple and maximal power according to the wind's speed can be deduced. For this, the mechanical characteristics of the turbine – WT and the orthogonal model of PMSG are used. The experimental characteristics of the WT were used to confirm the mathematical model proposed.

REFERENCES

1. BĂBESCU M., *Maşina Sincronă – Modelare – Identificare – Simulare* – ISBN 973-625-021-0, Editura POLITEHNICA Timişoara, 2003;
2. BEJ, A., *Turbine de vânt* – ISBN 973-625-098-9, Editura POLITEHNICA Timişoara,2003;
3. QIAO, W., HARLEY R. G., VENAYAGAMOORTHY G. K., *"Coordinated Reactive Power Control of a Large Wind Farm and a STATCOM Using Heuristic Dynamic Programming"*, IEEE Trans. Energy Convers., **vol. 24**, no. 2, pp.493-503 June 2009;
4. MAURICIO J. M., LEON A. E., GOMEZ-EXPOSITO A., SOLSONA J. A., *"An Adaptive Nonlinear Controler for DFIM – Based Wind Energy Conversion Systems"*, IEEE Trans. Energy Convers., **vol. 23**, no. 4, pp.1025-1035 December 2008;
5. FADAEINEDJAD R., MOSCHOPOULOS G., MOALLEM M., "*The Impact of Tower Shadow, Yaw Error, and Wind Shears on Power Quality in a Wind-Diesel System"*, IEEE Trans. Energy Convers., **vol. 24**, no. 1, pp.102-111, March 2009;
6. SHAHABI M., HAGHIFAM M. R., MOHAMADIAN M., NABAVI-NIAKI S. A., *"Microgrid Dynamic Performance Improvement Using a Doubly Fed Induction Wind Generator",* IEEE Trans. Energy Convers., **vol. 24**, no. 1, pp.137-145, March 2009;
7. MUHANDO E. B., SENJYU T., KINJO H., FUNABASHI T., *"Extending the Modeling Framework for Wind Generation Systems: RLS – Based Paradigm for Performance Under High Turbulence Inflow"*, IEEE Trans. Energy Convers., **vol. 24**, no. 1, pp.211-221, March 2009;
8. ULLAH N. R., BHATTACHARYA K., THIRINGER T., *"Wind Farms as Reactive Power Ancillary Service Providers-Technical and Economic Issues "*, IEEE Trans. Energy Convers., **vol. 24**, no. 3, pp.661-672, September 2009;
9. BARAKATI S. M., KAZERANI M., . APLEVICH J. D, *"Maximum Power Tracking Control for a Wind Turbine System Including a Matrix Converter* ", IEEE Trans. Energy Convers., **vol. 24**, no. 3, pp.705-713, September 2009;
10. TELEKE S., BARAN M. E., HUANG A. Q., BHATTACHARYA S., ANDERSON L., *"Control Strategies for Battery Storage for wind Farm Dispatching "*, IEEE Trans. Energy Convers., **vol. 24**, no. 3, pp.725-732, September 2009;
11. QUASCHNING V., *"Understanding Renewable Energy Systems ",* ISBN 1-84407-128-6, London Carl Hanser Verlang GmbH & Co KG, 2005.