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ASPECTS REGARDING THE WHEEL LOADS OF TRACTOR-SEMITRAILER ROAD TRAIN

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KEYWORDS

ground reactions, longitudinal dynamics, articulated tractor-trailer combination, braking force distribution, mathematic model

ABSTRACT

The article presents an algorithm that permits to calculate the wheel loads during straight-line travel for a tractor-trailer combination containing two articulated component vehicles. The algorithm can be used to predict the loads on axles, wheels or articulation pivot, to estimate the maximal attainable performances in longitudinal dynamics (acceleration, braking, climbing, downhill), to establish the best ratios between the axles' tractive or braking forces or to find the influence of each constructive or operating parameters that affect the loads.

Considering an example road train, some results are presented showing the use possibilities of the described method.

MAIN SECTION

PRELIMINARY ASSUMPTIONS

It is assumed a straight and even smooth road, i.e. the slope remains constant at an angle α and no bumps produce normal-load variations at the wheels.

The road train is considered as a combination of two rigid bodies – a tractor vehicle articulated with a (semi)trailer. In this situation the influences of the suspension or wheel deflections are cancelled out.

A mobile coordinate system (linked to the vehicle) is adopted according to ISO convention (with respect to the vehicle, the x axis is oriented forward, on the longitudinal plane of symmetry, y – lateral, out the left side of the vehicle, z – upward, normal to the ground).

A schematic drawing of a tractor-semitrailer combination presents in figure 1. Considering low and moderate speeds, the aerodynamic forces are neglected to simplify the formulae of calculus. Also neglects the rolling resistance moments. In this situation remain only the gravitational forces (G_t and G_s applied in the centres of gravity of the tractor and semitrailer) and the wheel loads (Z – normal ground reactions, perpendicular to the road plane; X – longitudinal ground reactions, in the road plane) acting on the middle of each wheel contact patch.

If tandem axles are used, the loads Z and X consider for all these axles (normally two or three). The applying point of these forces determines with respect to the suspension design, writing a supplementary equilibrium equation for the tandem moments. In the example of figure 1, the loads of trailer axles can be considered equal. Results that the forces Z_3 and X_3, as summation of respective forces of both trailer-axles, act at a point equal distanced of the axles.

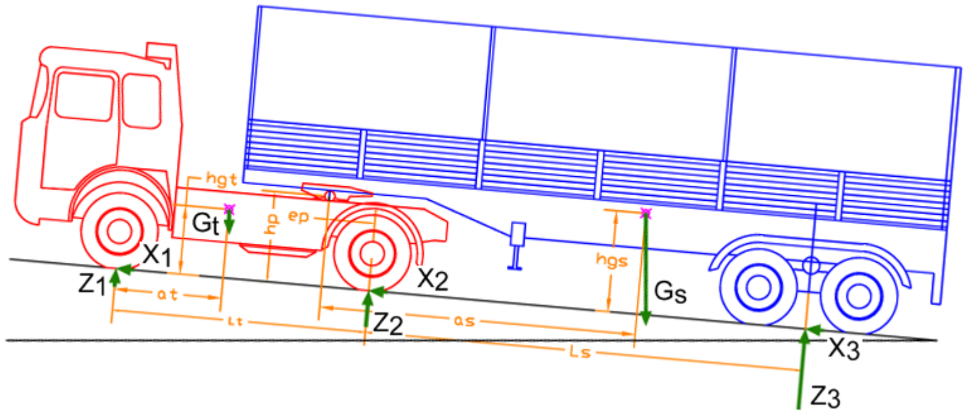

Fig. 1 Schema of forces acting on the road train during travel

The two bodies can be imaginarily separated at the articulation pivot (fifth wheel) but is necessary to replace the link with two pairs of forces Z_p and X_p (the first pair of forces load the tractor and the second sustain the semitrailer).

Because a planar movement was supposed for the road train, for each body (tractor and semitrailer) of the combination applies the Newton's second law for each degree of freedom (bounce – translation on z; fore-aft oscillation – translation on x; pitch – rotation on y) and obtains six equations. Because of the previous hypotheses, null values can be considered for the translational acceleration on z direction and the rotational acceleration on y direction. Also, assuming no relative displacement between tractor and semitrailer, the relative accelerations on x direction of the two bodies take the same value a_{xr} (the relative acceleration represents vehicle acceleration divided by gravitational acceleration or vehicle acceleration expressed in g).

Tree additional equations can be written if considers the actual value ξ of the friction coefficient for each axle. This coefficient is named also "used friction" and represents the ratio of tangential ground reaction X and normal ground reaction Z. In the ISO convention, positive friction coefficient associate with driving wheels and negative, with braking wheels. If the peak friction coefficient notes μ, then $\xi_{min} = -\mu \le \xi \le \mu = \xi_{max}$. For a free wheel (no tractive or braking torque applied) $\xi = -f$, where f is the rolling resistance coefficient.

The values ξ are limited not only by the quality of the wheel-ground interaction, but also by the driving or braking torque M applied to the axle wheels, depending on the engine, transmission or braking system performances. If the friction limits are not exceeded, the "used friction" is $\xi=(M/r_d)/Z$.

MATHEMATIC MODEL

According to figure 1 and the presented assumptions, the simplest mathematic model permitting to obtain the axle loads of an articulated tractor-trailer combination takes the shape of Eq. 1. The system contains nine equations and permits to find an equal number of unknowns. These nine unknowns can be chosen between thirteen quantities that can present interest: Z_1, Z_2, Z_3, X_1, X_2, X_3, Z_P, X_P, a_{xr}, α, ξ_1, ξ_2, ξ_3. To solve the system of equations it is necessary to specify values for four such quantities, normally between a_{xr}, α, ξ_1, ξ_2, ξ_3.

$$Z1 + Z2 - Z_p - G_t \cdot \cos(\alpha) = 0$$
$$Z3 + Z_p - G_s \cdot \cos(\alpha) = 0$$
$$X1 + X2 - G_t \cdot \sin(\alpha) - X_p = G_t \cdot a_{xr}$$
$$X3 - G_s \cdot \sin(\alpha) + X_p = G_s \cdot a_{xr}$$
$$Z1 \cdot a_t - Z2 \cdot (L_t - a_t) + Z_p \cdot (L_t - a_t - e_p) - X_p \cdot (hg_t - h_p) + (X1 + X2) \cdot hg_t = 0$$
$$Z_p \cdot a_s - Z3 \cdot (L_s - a_s) + X_p \cdot (hg_s - h_p) + X3 \cdot hg_s = 0$$
$$X1 = \xi 1 \cdot Z1$$
$$X2 = \xi 2 \cdot Z2$$
$$X3 = \xi 3 \cdot Z3$$

Eq. 1

Using a computer program, the numerical solving of this system presents no difficulty. Some mathematical programs can also perform a direct symbolic solving, but the expressions of the solution are to long and, for that, impractical to use. To simplify the expressions, in this work was preferred the substitution method and some auxiliary expressions were used. First, makes the notations in Eq. 2

$$b_t = L_t - a_t \qquad b_s = L_s - a_s \qquad b_p = b_t - e_p$$
$$h_{tp} = hg_t - h_p \qquad h_{sp} = hg_s - h_p \qquad G_a = G_t + G_s$$

Eq. 2

and eliminates the unknowns ξ_1, ξ_2, ξ_3.

The auxiliary expressions

$$E1 = b_p + a_t - \xi 3 \cdot \frac{G_t}{G_a} \cdot h_{tp} + \xi 1 \cdot \left(hg_t - \frac{G_s}{G_a} \cdot h_{tp} \right)$$
$$E2 = b_p - b_t - \xi 3 \cdot \frac{G_t}{G_a} \cdot h_{tp} + \xi 2 \cdot \left(hg_t - \frac{G_s}{G_a} \cdot h_{tp} \right)$$
$$E3 = L_s + \left(\xi 3 \cdot \frac{G_t}{G_a} + \xi 1 \cdot \frac{G_s}{G_a} \right) \cdot h_{sp} - \xi 3 \cdot hg_s$$
$$E4 = L_s + \left(\xi 3 \cdot \frac{G_t}{G_a} + \xi 2 \cdot \frac{G_s}{G_a} \right) \cdot h_{sp} - \xi 3 \cdot hg_s$$
$$E5 = G_t \cdot (b_p - \xi 3 \cdot h_{tp})$$
$$E6 = G_t \cdot \xi 3 \cdot h_{sp} - G_s \cdot a_s + G_a \cdot (L_s - \xi 3 \cdot hg_s)$$
$$E7 = E1 \cdot E4 - E2 \cdot E3$$

Eq. 3

are functions of axles' used friction ξ_1, ξ_2, ξ_3, but are not depending on the road slope α. These expressions have dimensions of length (E1…E4), torsional moment (E5, E6) and surface (E7).

$$Z1 = \frac{(E4 \cdot E5 - E2 \cdot E6)}{E7} \cdot \cos(\alpha) \qquad X1 = \xi 1 \cdot Z1$$
$$Z2 = \frac{(E1 \cdot E6 - E3 \cdot E5)}{E7} \cdot \cos(\alpha) \qquad X2 = \xi 2 \cdot Z2$$
$$Z3 = G_a \cdot \cos(\alpha) - Z1 - Z2 \qquad X3 = \xi 3 \cdot Z3$$
$$Z_p = G_s \cdot \cos(\alpha) - Z3 \qquad X_p = \frac{(X1 + X2) \cdot G_s - X3 \cdot G_t}{G_a}$$
$$a_{xr} = \frac{X1 + X2 + X3}{G_a} - \sin(\alpha)$$

Eq. 4

The definition of a_{xr} in Eq. 4 can be used to find the extreme road slope (maximal or minimal) that the motor vehicle can climb or descend with constant velocity when are imposed the values of axles' used friction. Making $a_{xr}=0$, obtains:

$$\alpha_{ex} = \operatorname{asin}\left(\frac{X1 + X2 + X3}{G_a}\right) = \operatorname{asin}\left(\frac{\xi 1 \cdot Z1 + \xi 2 \cdot Z2 + \xi 3 \cdot Z3}{G_a}\right) \qquad \textit{Eq. 5}$$

If the "friction is equally used" by all axles ($\xi_1 = \xi_2 = \xi_3 = \xi$)

$$\sin(\alpha_{ex}) = \frac{\xi \cdot Z1 + \xi \cdot Z2 + \xi \cdot Z3}{G_a} = \xi \cdot \frac{Z1 + Z2 + Z3}{G_a} = \xi \cdot \frac{G_a \cdot \cos(\alpha)}{G_a} = \xi \cdot \cos(\alpha_{ex}) \qquad \textit{Eq. 6}$$

and obtains the maximal grade α_{max} to be mounted by an all-wheel-drive vehicle and the extreme slope α_{min} that can be descended with constant velocity by an all-wheel-braked vehicle:

$$\alpha_{max} = \operatorname{atan}(\mu) \qquad \alpha_{min} = -\operatorname{atan}(\mu) \qquad \textit{Eq. 7}$$

Sometimes, for example to study the performances of a existent braking system or of a multi-axle drive train using open inter-axle differentials, the values of the used friction are unknown, but are imposed some ratios (k_2, k_3) between axles' longitudinal forces. In that case the system in Eq. 1 takes the form

$$\begin{aligned}
&Z1 + Z2 - Z_p - G_t \cdot \cos(\alpha) = 0 \\
&Z3 + Z_p - G_s \cdot \cos(\alpha) = 0 \\
&Z1 \cdot a_t - Z2 \cdot (L_t - a_t) + Z_p \cdot (L_t - a_t - e_p) - X_p \cdot (hg_t - h_p) + (X1 + X2) \cdot hg_t = 0 \\
&Z_p \cdot a_s - Z3 \cdot (L_s - a_s) + X_p \cdot (hg_s - h_p) + X3 \cdot hg_s = 0 \\
&X1 + X2 - G_t \cdot \sin(\alpha) - X_p = G_t \cdot a_{xr} \\
&X3 - G_s \cdot \sin(\alpha) + X_p = G_s \cdot a_{xr} \\
&X2 = X1 \cdot k2 \\
&X3 = X1 \cdot k3
\end{aligned} \qquad \textit{Eq. 8}$$

Considering known X_1 and α, the eight equations permits to find eight unknown: Z_1, Z_2, Z_3, X_2, X_3, Z_P, X_P, a_{xr}. The solution for this system is:

$$k_p = \frac{\left[(1 + k2) \cdot G_s - k3 \cdot G_t\right]}{G_a} \qquad k_h = k3 \cdot hg_s + k_p \cdot h_{sp} \qquad E8 = \frac{G_s \cdot a_s \cdot \cos(\alpha) + X1 \cdot k_h}{L_s}$$

$$X2 = X1 \cdot k2 \qquad X3 = X1 \cdot k3$$

$$X_p = X1 \cdot k_p \qquad Z_p = G_s \cdot \cos(\alpha) - Z3$$

$$Z1 = \frac{E8 \cdot (b_p - b_t) + (G_a \cdot b_t - G_s \cdot b_p) \cdot \cos(\alpha) - X1 \cdot \left[(1 + k2) \cdot hg_t - k_p \cdot h_{tp}\right]}{L_t}$$

$$Z2 = G_a \cdot \cos(\alpha) - E8 - Z1 \qquad Z3 = \frac{G_s \cdot a_s \cdot \cos(\alpha) + X1 \cdot k_h}{L_s}$$

$$a_{xr} = \frac{X1}{G_a} \cdot (1 + k2 + k3) - \sin(\alpha) \qquad \textit{Eq. 9}$$

with three auxiliary notations: k_p, k_h, *E8*.

After that can be also computed the real used frictions ξ_1, ξ_2, ξ_3, using the last three definitions of Eq. 1.

The extreme slope that the road train can climb or descend with a constant velocity obtains making a_{xr}=0 in the last definition of Eq. 9, resulting

$$\alpha_{ex} = \mathrm{asin}\left[\frac{X1}{G_a}\cdot(1 + k2 + k3)\right] \qquad \textit{Eq. 10}$$

ALGORITHM USING EXAMPLE

As an example, a real tractor-semitrailer combination was considered. The data necessary for computation is: m_a=38000 kg, m_t=6900 kg, L_t=3.5 m, L_s=6.085 m, a_t=1.487 m, a_s=4.383 m, e_p=0.656 m, h_p=1.25 m. The road train's traction layout is a classical one, with only one drive axle – the tractor's rear axle. In these case, the used friction of tractor front axle and semitrailer axles are $\xi_1=\xi_3=-f$.

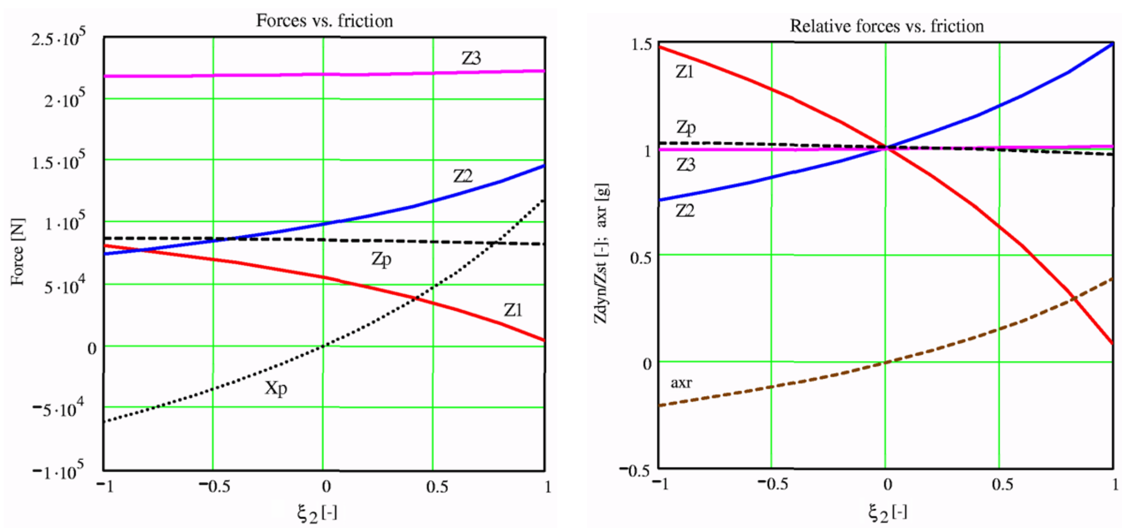

Fig. 2 Tractor rear-wheel drive and tractor rear-wheel (engine or retarder) braking
left – magnitudes of axles- and pivot-forces; right – ratios of dynamic and static forces, ratio of vehicle longitudinal acceleration and gravitational acceleration

Figure 2 shows the way the axles' normal forces (Z_1, Z_2, Z_3) and the pivot's forces (Z_p, X_p) depend on the used friction of the tractor's rear axle (ξ_2) when a horizontal ground is assumed. Positive ξ_2 values correspond to traction and negative values to engine or retarder braking.

In the left-side plot are presented the dynamic values of forces and in the right-side these values are divided by the static loads. As can be seen, the dynamic loads Z_3 of the semitrailer axles and the normal load Z_p of the pivot change slightly, but dramatic modifications suffer the dynamic loads of tractor axles – extreme traction increases rear-axle's load with 50% and decreases rear-axle's load with 90%, making the road train almost unsteerable. Important changes, in an opposite way, happen when the tractor's rear axle brakes intensely.

The a_{xr} curve in the right-side plot shows also the obtainable accelerations on horizontal roads with different peak friction coefficients. For example, on a road with μ=0.8, an acceleration of 0.28 g and a deceleration of 0.18 g can be obtained applying tractive, respective braking moments, on the tractor's rear axle.

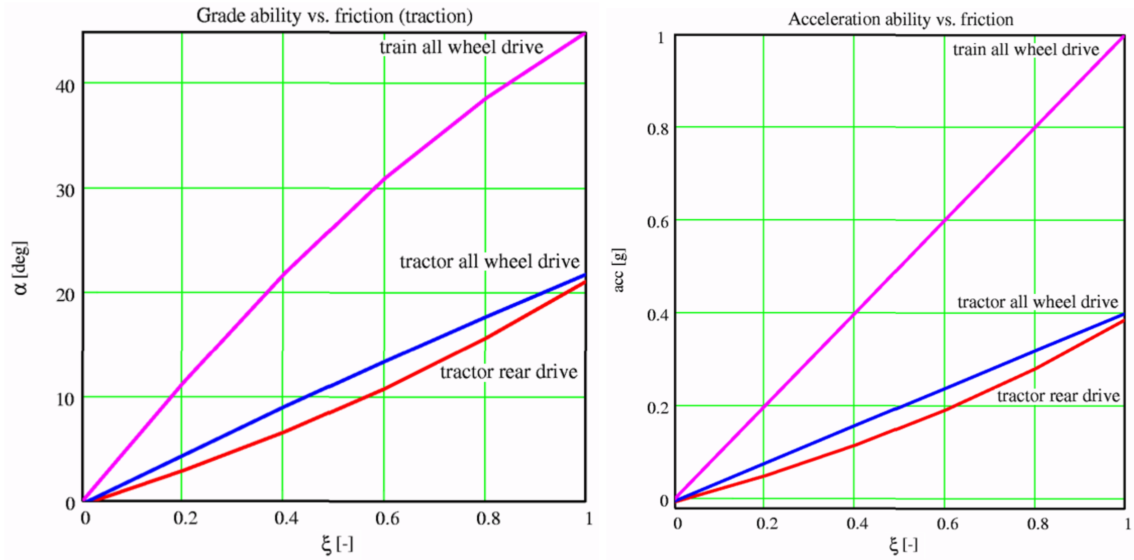

Fig. 3 Road train grade ability (left) and acceleration ability (right) for different traction formulae

In the left side of figure 3 is represented the road train's climbing capability. Three different driving cases were considered. The best performances obtain if all wheels of tractor and semitrailer are driving ($\xi_1=\xi_2=\xi_3=\xi$): for a road with peak friction coefficient μ=0.8, the road train can ascend a maximal slope of 38°. The performances decrease dramatically if only the tractor wheels are driving ($\xi_1=\xi_2=\xi$, $\xi_3=-f=0.01$) and even more if a tractor rear-wheel drive formula is used ($\xi_2=\xi$, $\xi_1=\xi_3=-f$) – for μ=0.8 results 18°, respectively 15°. Comparing the classical tractor rear-wheel drive formula with tractor all-wheel drive formula finds that the traction improvement is not so big (as the increase of drivetrain complexity) and this diminishes at high friction coefficients, due to the front axle unloading.

The right side of figure 3 presents the accelerations variations in the case of a horizontal road and the same traction formulae. For a road with μ=0.8, the maximal obtainable accelerations are respectively: 0.8 g, 0.32 g and 0.28 g. However, the plotted values can be reached only if the drivetrain distributes the driving forces (driving torques divided by respective wheel radii) in the right ratios, these changing continuously as functions of used friction.

The presented algorithm proves extremely useful for the study of the road-train braking ability. During a service braking, when all wheels must to contribute to decelerate the vehicle, maximal efficiency obtains if "the friction is equally used" by each axle ($\xi_1=\xi_2=\xi_3=\xi$). Normally, pushing the braking pedal, the driver controls the braking force X_1 of the tractor's front axle. For that reason it is preferable to represent the variation of the other longitudinal forces (X_2, X_3, X_p) versus X_1. To avoid the measurement units, in figure 4 are presented the forces divided by the fully-loaded train's gross weight G_a, considering friction coefficients $0>\xi>-0.8$. These ideal relative forces were represented for three extreme cases: fully-loaded train; unloaded train (no cargo); tractor only (without semitrailer). As the plot shows, to realise maximal deceleration on a given road, the vehicle braking system must largely modifying the forces magnitudes, not only with respect to the braking coefficient but also to the cargo's load, height or disposal.

Such a force regulation is practically impossible to achieve by the classical pneumatic-actuation systems (which can only approximate the plotted curves by straight lines). In addition, the lines slops must reduce if less- or no-cargo is carried and even more if the tractor travel without semitrailer. An optimal braking force distribution can be obtained only if electronic systems, including load sensors, are used. Even so, it is very difficult to maintain equal braking coefficients to all axles, due to the inherent road friction variations.

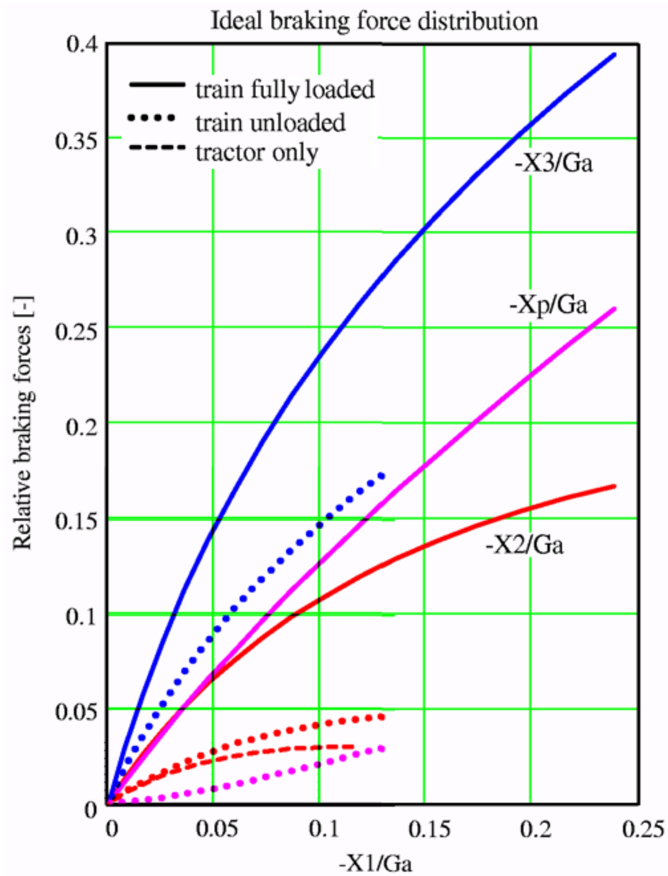

Fig. 4 Ideal longitudinal forces vs. tractor front-axle braking-force

For that reason, to avoid the most dangerous loss-of-stability situations ("jack-knife" effect, when tractor rear wheels lock first, or semitrailer "sweeping", when semitrailer wheels lock first), international standards regarding braking impose that $\xi_1 > \xi_2 > \xi_3$ for $\mu < 0.8$. That means that the real X_2 and X_3 braking forces being under the ideal curves in figure 4.

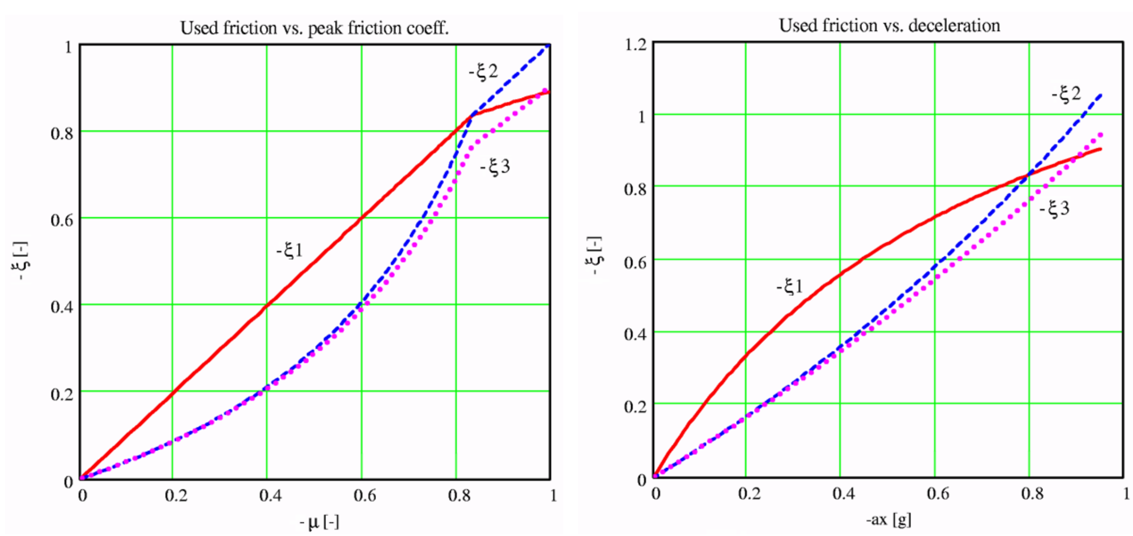

Fig. 5 Axles' used-friction real distribution

Considering, for the road train fully loaded, constant ratios X_2/X_1=0.667 and X_3/X_1=1.5 that respect the requirements of the braking standards, figure 5 shows the way that the axles' used friction depend on road peak friction coefficient and on vehicle deceleration. If ξ_3 adopted not so far of ξ_2 when μ=0.8 (in figure $\xi_3 \approx 0.9\ \xi_2$) to obtain good braking efficiency on dry asphalt or concrete roads, for low and medium friction values the two curves practically superpose, meaning that the danger of semitrailer "sweeping" remains high, for example on wet or dirty asphalt road. That means that other electronic braking device, the ABS, can be extremely helpful to maintain vehicle stability.

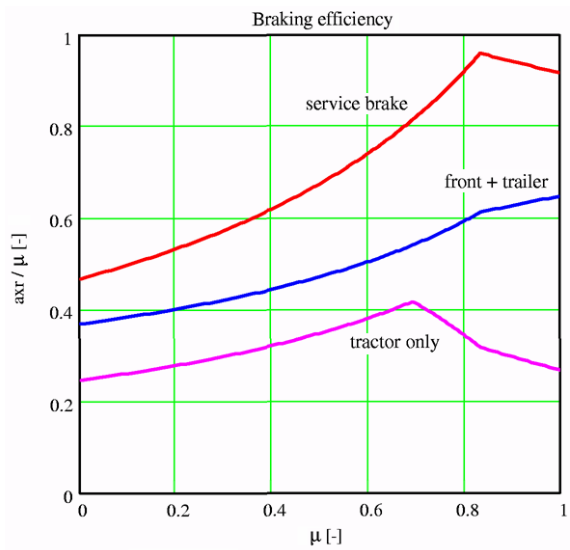

Fig. 6 Real braking efficiency for road train: service braking and two auxiliary braking cases

The quality of braking force distribution can be appreciated with the so called "braking efficiency" defined as the ratio of maximal relative deceleration and road's peak friction coefficient:

$$\eta_B = a_{xr} / \mu = (a_x / g) / \mu \qquad \textit{Eq. 11}$$

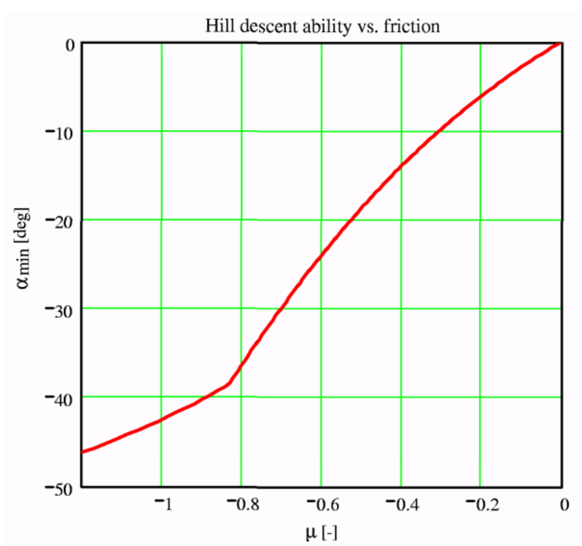

Fig. 7 Road train downhill ability considering the real braking distribution

Figure 6 presents this ratio as a function of peak friction coefficient μ for three different cases. The first case, that is also the best, corresponds to a service braking (with all axles). The others cases are considering that one-axle's commanding circuit is out of order and it realises auxiliary braking on others axles: tractor's front and semitrailer's axles; only tractor's axles.

Accordingly to the braking standards, the efficiency for both service braking and auxiliary braking must be greater than a certain level, depending on μ.

The last possibility of the algorithm presented here consists in the computation of the extreme road slope that can be descended with constant speed by the road train. Figure 7 shows the inclination of the road that can be descended when all the wheels brakes are applied as a function of the peak friction coefficient.

CONCLUSIONS

The presented algorithm permits to compute the magnitude of forces acting an articulated tractor-trailer combination being in a straight-line movement. The algorithm can be used to:

- predict the loads on axles, wheels or articulation pivot;
- estimate the maximal longitudinal dynamics performances (acceleration, braking, climbing, downhill);
- establish the best ratios between the axles' tractive or braking forces;
- find the influence of each constructive (wheelbase; pivot position; centre of gravity) or operating (road friction; load weight, height or position) parameters that affect the loads.

The algorithm can be also used for other vehicle categories – for example, car with one axle trailer or articulated bus. The only care is to use a negative e_p value to indicate an articulation point behind the tractor rear axle. Also, making G_s=0, the algorithm applies with no modification for single vehicles.

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