5th International Conference
"Computational Mechanics and Virtual Engineering"
COMEC 2013
24- 25 October 2013, Braşov, Romania

RANDOM VIBRATION FOR THE DISK-SHAFT SYSTEMS WITH TWO DEGREES OF FREEDOM

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Abstract: *Random vibrations are extensively used in transportation, wind and earthquake. Exact solutions for random excitations are very limited, particulary when the material behavoir is hysteretic, having a multi-value forced deformation pattern with non conservative energy dissipation. Also information about the need mathematical apparatus is included. Formulation of the equivalent linearization has been used to analyze system using differential mathematical models with approximed solutions. In equivalent linearization method the governing set of nonlinear differential equations are replaced by an equivalent set of linear equations, and the difference between the sets being minimized in some appropriate sense. The results obtained from the method are validated by simulation results.*
Keyword**:** *Nonlinear systems, ramdom vibrations, vibration analysis, linear equation.*

1. SYSTEM MODEL

Consider the system composed of two disks with moments polar mass inertia J [$kg\cdot m^2$] mounted on shafts torsional rigidity K1, K2, [Nm rad] with negligible mass.
We consider the angles θ_1 and θ_2 snapshots of the disks in compared with the the position of static equilibrium and $M(t)$ is the random cuple. Using the principle of d'Alembert [1,3] (dynamic equilibrium of external torques and inertia), equation of the motion are written in the following form

$$\begin{pmatrix} J & 0 \\ 0 & J \end{pmatrix}\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix}+\begin{pmatrix} c & -c \\ -c & c \end{pmatrix}\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}+\begin{pmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2 \end{pmatrix}\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}+\beta\begin{pmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2 \end{pmatrix}\begin{pmatrix} \theta_1^3 \\ \theta_2^3 \end{pmatrix}=\begin{pmatrix} M(t) \\ 0 \end{pmatrix}. \tag{1}$$

The linear equation [2] can be write

$$\begin{pmatrix} J & 0 \\ 0 & J \end{pmatrix}\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix}+\begin{pmatrix} c_e & -c_e \\ -c_e & c_e \end{pmatrix}\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{pmatrix}+\beta\begin{pmatrix} K_{1e}+K_{2e} & -K_{2e} \\ -K_{2e} & K_{2e} \end{pmatrix}\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}=\begin{pmatrix} M(t) \\ 0 \end{pmatrix} \tag{2}$$

The nonlinear factor β controls the type and degree of nonlinearity in the system.
The difference between the nonlinear stiffness and linear stiffness terms is

$$e=\begin{pmatrix} e_1 \\ e_2 \end{pmatrix}=\begin{pmatrix} (K_1+K_2)(\theta_1+\beta\theta^3{}_1)-K_2(\theta_2+\beta\theta^3{}_2)-(K_{1e}+K_{2e})\theta_1+k_{2e}\theta_2 \\ -K_2(\theta_1+\beta\theta^3{}_1)+K_2(\theta_2+\beta\theta^3{}_2)-K_{2e}\theta_2+K_{2e}\theta_1 \end{pmatrix}. \tag{3}$$

The value of K_{1e} and K_{2e} can be obtained by minimizing [2] the expectation of the square error

$$\frac{dE\{e_1{}^2\}}{dK_{1e}}=0, \tag{4}$$

and

$$\frac{dE\{e_2{}^2\}}{dK_{2e}}=0. \tag{5}$$

Neglecting wery small terms we get

$$K_{1e}=K_1+3\beta\sigma_{\theta_1}^2\left(K_1+K_2\right)-\frac{3\alpha K_2\left(\sigma_{\theta_1}^4+\sigma_{\theta_2}^4\right)}{\sigma_{\theta_1}^2+\sigma_{\theta_2}^2}, \tag{6}$$

$$k_{2e}=k_2+\frac{3\alpha k_2\left(\sigma_{\eta_1}^4+\sigma_{\eta_2}^4\right)}{\sigma_{\eta_1}^2+\sigma_{\eta_2}^2}. \tag{7}$$

Using the Fourier transform of equation [1,3] and having the relations

$$\mathrm{F}\,(\ddot{\theta}(t))=-\omega^2\,\bar{\theta}\,(\omega) \tag{8}$$

$$\mathrm{F}\,(M(t))=\bar{M}(\omega), \tag{9}$$

obtain for the response

$$\bar{\theta}_1(\omega)=\frac{J^2\omega^2-2J_1K_{2e}-2i\omega cJ}{D}\bar{M}(\omega), \tag{10}$$

$$\bar{\theta}_2(\omega)=\frac{J^2\omega^2-J(K_{2e}+K_{1e}+K_{2e})K_{2e}-3i\omega cJ}{D}\bar{M}(\omega), \tag{11}$$

where

$$D=J^2\omega^4-\omega^2J(K_{2e}+K_{1e}+K_{2e})+K_{1e}K_{2e}. \tag{12}$$

The frequency response function [5,6] of the system is give by equation

$$\bar{H}_1(\omega)=\frac{J^2\omega^2-2JK_{2e}}{D} \tag{13}$$

and

$$\bar{H}_2(\omega)=\frac{J^2\omega^2-J(K_{2e}+K_{1e}+K_{2e})}{D}. \tag{14}$$

The mean square value for the displacement [1,2] of the system is given by equation

$$\sigma^2_{\theta_1}=R_{\eta_1}(0)=\int_{-\infty}^{\infty}\left|H_1(\omega)\right|^2J^2S'_0d\omega=\frac{1}{J_1^2}\int_{-\infty}^{\infty}\left|\bar{H}_1(\omega)\right|^2J^2S'_0d\omega=S'_0\int_{-\infty}^{\infty}\frac{[J^2\omega^2-2Jk_{2e}]^2+4J^2\omega^2c^2}{m^2+n^2}d\omega \tag{15}$$

and for the second structure

$$\sigma^2_{\theta_2}=R_{\theta_2}(0)=\int_{-\infty}^{\infty}\left|\bar{H}_2(\omega)\right|^2S'_0d\omega=S'_0\int_{-\infty}^{\infty}\frac{[J^2\omega^2-J^2(K_{2e}+K_{1e}+K_{2e})]^2+9c^2\omega^2J^2}{m^2+n^2}d\omega. \tag{16}$$

where

$$m=J^2\omega^4-\omega^2(2Jk_{2e}+Jk_{1e}+3c^2)+k_{1e}k_{2e}, \tag{17}$$

$$n=\omega\;c(5k_{2e}+k_{1e})-3Jc\omega^3 \tag{18}$$

Obtain for the first disk

$$\sigma^2_{\eta_1}=\pi S'_0\frac{F}{G}, \tag{19}$$

and for the second disk

$$\sigma^2_{\theta_2}=\pi S'_0\frac{H}{G}, \tag{20}$$

where

$$F=\frac{4K_{2e}J^2}{K_{1e}}[3(2JK_{2e}+JK_{1e}+3c^2)Jc-(5K_{2e}+K_{1e})J^2c]+3Jc[4c^2J^2+4K_{2e}J^3]+cJ^4(5K_{2e}+K_{1e}) \tag{21}$$

$$G=c(5K_{2e}+K_{1e})[3(2JK_{2e}+JK_{1e}+3c^2)Jc-(5K_{2e}+K_{1e})J^2c]-9K_{1e}K_{2e}J^2c^2 \tag{22}$$

$$H=\frac{4K_{2e}J^2}{K_{1e}}[3(2JK_{2e}+JK_{1e}+3c^2)Jc-(5K_{2e}+K_{1e})J^2c]+3Jc[9J^2c^2-2J(2K_{2e}+K_{1e})]+J^4c^2(5K_{2e}+K_{1e})^2 \tag{23}$$

The power spectral density of response [1,6,7] for the first disk is given by equation

$$S_1(\omega)=\left|H_1(\omega)\right|^2S_M=\left|H_1(\omega)\right|^2J^2S'_0(\omega)=\left|\frac{1}{J}\bar{H}_1(\omega)\right|^2J^2S'_0(\omega)=\left|\bar{H}_1(\omega)\right|^2S'_0(\omega), \tag{24}$$

and the power spectral density for the second structure (in $rad^2 \cdot s$) is given by equation

$$S_2(\omega)=\left|\bar{H}_2(\omega)\right|^2 S'_0(\omega)\,, \tag{25}$$

So, the power spectral density of response will be

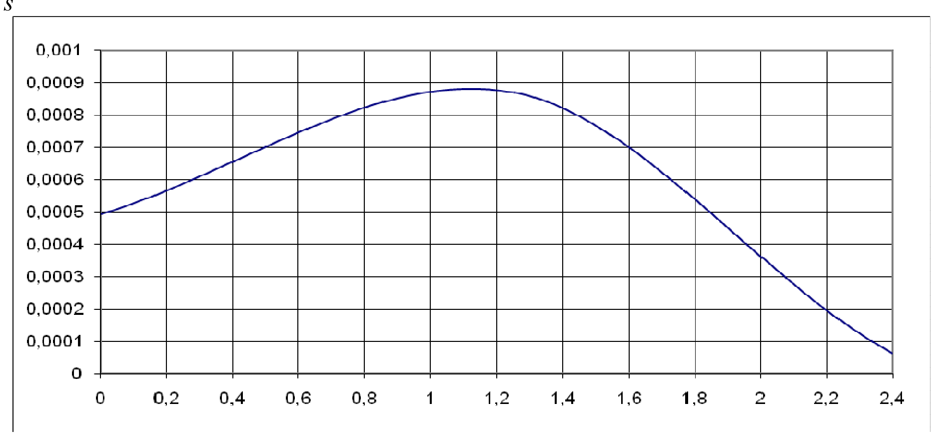
$$S_1(\omega)=\frac{[J^2\omega^2-2JK_{2e}]^2+4J^2c^2\omega^2}{m^2+n^2}S'_0(\omega)\,, \tag{26}$$

$$S_2(\omega)=\frac{[J^2\omega^2-(2JK_{2e}+JK_{1e})]^2+9J^2c^2\omega^2}{m^2+n^2}S'_0(\omega)\,. \tag{27}$$

2. NUMERICAL RESULTS

As a numerical example, a disk-shaft systems with systems with two degrees driven by both narrow- and wide-band excitations is considered. The results of the study show the following.

Considered $J=50kg\cdot m^2$, $K_1=K_2=7\cdot10^6\ N\cdot m/rad$, $S_M=1\,N^2\cdot s$ by four identical columns of Young's modulus $E=0,2\cdot10^{11}Pa$ and height $h=2m$, with the diameter $d=0,5m$, the damping factor $\xi=0,25$, the nonlinear component $G(\eta)=\eta^3(t)$, with the nonlinear factor to control the type and degree of nonlinearity $\alpha=20m^{-2}$, and $S'_0=0,52\frac{rad^2}{s^3}$, wich means that the power spectral density of excitation $S_F=m^2S'_0=2,08\cdot10^{10}N^2\cdot s$.



$$\frac{\omega}{2\pi}$$

Fig.1. The power spectral density of the response $S_1(\omega)\ [rad^2\cdot s]$ for the first disk.

The value of K_{1e} and K_{2e} are

$$K_{1e}=7,315\cdot10^6\ N\cdot m/rad,\ K_{2e}=7,572\cdot10^6\ N\cdot m/rad \tag{28}$$

The power spectral density of the response for the first disk, in this case, is

$$S_1(\omega)=\frac{(J\omega^2-2K_{2e})^2+4\omega^2c^2}{m^2+n^2}S_M\,,\quad [m^2\cdot s] \tag{29}$$

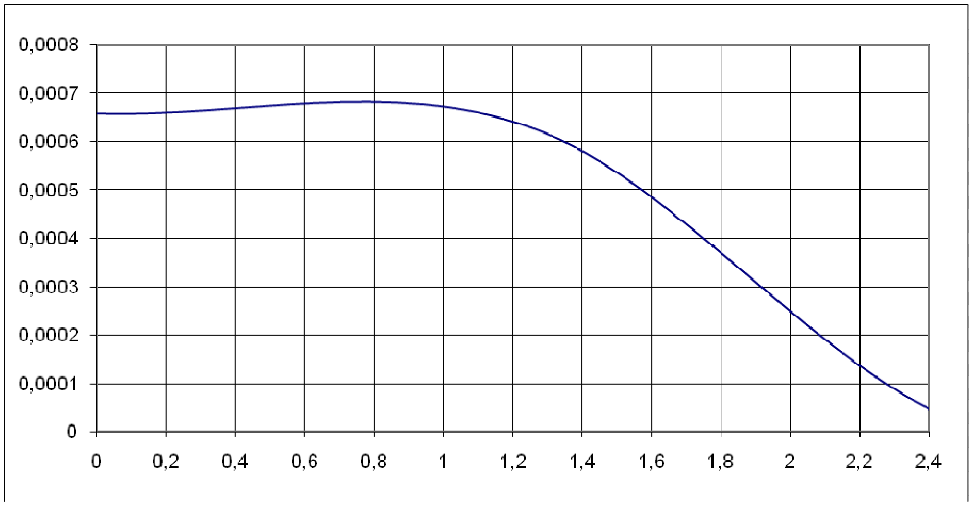
and the power spectral density of the response for the second disk is

$$S_2(\omega)=\frac{[J\omega^2-(2K_{2e}+K_{1e})]^2+9\omega^2c^2}{m^2+n^2}S_M\,,\quad [m^2\cdot s] \tag{30}$$

where

$$m=J^2\omega^4-\omega^2(2Jk_{2e}+Jk_{1e}+3c^2)+k_{1e}k_{2e}\,, \tag{31}$$

$$n=\omega\ c(5k_{2e}+k_{1e})-3Jc\omega^3 \tag{32}$$



$$\frac{\omega}{2\pi}$$

Fig.2. The power spectral density of the response $S_2(\omega)$ $[rad^2 \cdot s]$ for the second disk.

CONCLUSIONS

The method has been proposed for determinate the power spectral response is based on considering the non-linear system to behave as a linear system having varyng natural frequency.

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