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APPLICATION OF THE GALERKIN-VLASOV VARIATIONAL METHOD IN THE STUDY OF FREE VIBRATIONS OF THE SQUARE PLATE C-C-SF-F

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Abstract: *Initially, in the case of rectangular plate it was considered that is sufficient be taken into account only the resonance phenomenon. Later it being understood that vibration systems located away from the resonance can even modify and alter the structure of the materials. The literature reflected that the assessment respectively static dynamic response of flat plate is done taking into account the shape of the median surface, boundary conditions, loading mode and solving method adopted.*

Key words: ***plate, variational, free, vibrations***.

1. Introduction

Calculation of flat plate to solve the problem of free and forced vibration is reflected in the literature with contributions by several authors such as: Warburton G. B, [19], [20], which presented a set of solutions for the six cases of rectangular plates, Janich R., resulted in [12] a complete set of solutions for 18 of the 21 possible combinations of boundary conditions, S. Iguchi [9], H. Fletcher [7], whose concerns were directed to solve free vibration using semianalytic methods, respectively M. Hamada [10], which has studied the plates by variational methods.

2. Objectives

It is known that most of the complications that appear in flat plates solving are related to the existence of free sides [11], [12]. These difficulties are reflected by the impossibility of finding functions to describe the stress in the plate having rigorous static conditions on free side [4], [14]. Scope of this paper is represented by the study of free vibration for rectangular plate clamped on two opposite sides and simply supported and free on the other two (SS-F-C-C).

3. Materials and Methods

Flat plate analyzed is considered as being thin, elastic, isotropic with bending stiffness and meets the conditions of validity of Kirchhoff's hypothesis [4], [5], [11], [18]. The proposed calculation method is an adaptation of variational method Galerkin-Vlasov [5], [16], [17], for static flat plates and was so elaborate that the calculations necessary to determine the characteristics of the plate are made on the

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basis of programs prepared by the author [5]. The method can be regarded as a Bubnov-Galerkin method particularization, differing from it by the fact that the determination of stress and displacements are not considered the disturbance efforts resulting from imperfection functions chosen to approximate displacements. Is considered a rectangular plate presented in figure 1, with dimensions a and b clamped in the sides $x = 0, x = a$, simply supported on the side $y = 0$ and free on the side $y = b$.

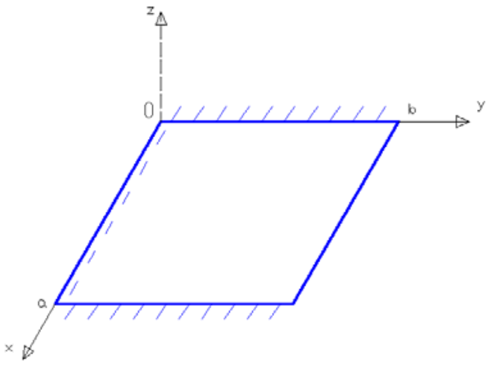

Fig. 1. Median surface of plate

On the median surface is considered a network in which nodes are determined the natural vibration shapes function values [5].

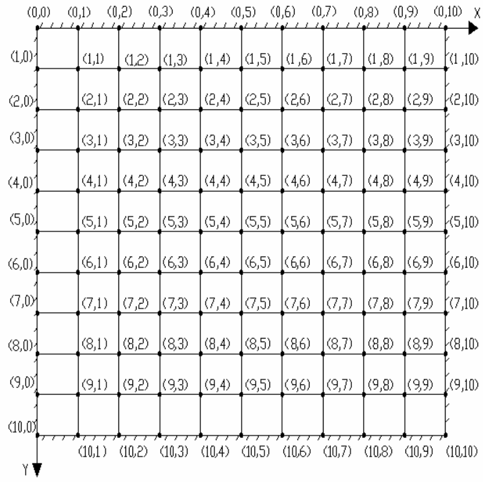

Fig.2 Median surface network

The expressions of beams shapes functions of vibration on the X and Y directions are [3]:

$$G_i(x) = \left(\cosh \beta_i \frac{x}{a} - \cos \beta_i \frac{x}{a}\right) - k_i \left(\sinh \beta_i \frac{x}{a} - \sin \beta_i \frac{x}{a}\right)$$

$$F_1(y) = \sqrt{3}\frac{y}{b} \qquad (1)$$

$$F_j(y) = \left(\sinh \beta_j \frac{y}{b} + \sin \beta_j \frac{y}{b}\right) - k_j \left(\sinh \beta_j \frac{y}{b} - \sin \beta_j \frac{y}{b}\right)$$

The parameter values $\beta_i, k_i, \beta_j, k_j$, were determined [3].

The expressions of shapes functions [1], [2], [3] for the plate are

$$\Phi_{ij}(x,y) = G_i(x) \cdot F_j(y) \qquad (2)$$

Sturm-Liouville problem [3] associated with flat square plate is:

$$\nabla^4 \Phi_{ij}(x,y) = \lambda_{ij} \Phi_{ij}(x,y)$$

$$\Phi_{ij}(0,y) = 0, \frac{\partial \Phi_{ij}}{\partial x}(0,y) = 0$$

$$\Phi_{ij}(a,y) = 0, \frac{\partial \Phi_{ij}}{\partial x}(a,y) = 0$$

$$\Phi_{ij}(x,0)=0,\frac{\partial^2\Phi_{ij}}{\partial y^2}(x,0)=0$$

$$\frac{\Phi^2{}_{ij}}{\partial y^2}(x,b)+\nu\frac{\partial^2\Phi_{ij}}{\partial x^2}(x,b)=0$$

$$\frac{\partial^3\Phi_{ij}}{\partial y^3}(x,b)+(2-\nu)\frac{\partial^3\Phi_{ij}}{\partial x^2\partial y}(x,b)=0$$

Substituting equation functions of normal modes through their functions of beams products, integrating relationships across the plate and using the method of separation of variables is obtain [6], [15], [16]

$$\left(\frac{\beta_i}{a}\right)^4\int_0^a(G_i^{IV}(x)dx\cdot\int_0^b F_j^2(y)dy+2\left(\frac{\beta_i}{a}\right)^2\left(\frac{\beta_j}{b}\right)^2\int_0^a G_i''(x)G_i(x)dx\cdot\int_0^b F_j''(y)F_j(y)dy+$$

$$+\left(\frac{\beta_j}{b}\right)^4\int_0^a G_i^2(x)dx\cdot\int_0^b F_j^{IV}(y)F_j(y)=\lambda_{ij}\int_0^a G_i^2(x)dx\cdot\int_0^b F_j^2(y)dy \tag{3}$$

The expression of the pulsations parameters is [5]:

$$\lambda_{ij}=\frac{\left[\begin{array}{l}\left(\frac{\beta_i}{a}\right)^4\int_0^a(G_i^{IV}(x)dx\cdot\int_0^b F_j^2(y)dy+2\left(\frac{\beta_i}{a}\right)^2\left(\frac{\beta_j}{b}\right)^2\int_0^a G_i''(x)\cdot G_i(x)dx\cdot\int_0^b F_j''(y)F_j(y)dy+\\+\left(\frac{\beta_j}{b}\right)^4\int_0^a G_i^2(x)dx\cdot\int_0^b F_j^{IV}(y)F_j(y)\end{array}\right]}{\int_0^a G_i^2(x)dx\cdot\int_0^b F_j^2(y)dy} \tag{4}$$

We adopt the notations [5], [16]:

$$I_1=\left(\frac{\beta_i}{a}\right)^4\int_0^a[(\cosh\beta_i\frac{x}{a}-\cos\beta_i\frac{x}{a})-k_1(\sinh\beta_i\frac{x}{a}-\sin\beta_i\frac{x}{a})]^{IV}$$

$$\cdot\left[\left(\cosh\beta_i\frac{x}{a}-\cos\beta_i\frac{x}{a}\right)-k_i\left(\sinh\beta_i\frac{x}{a}-\sin\beta_i\frac{x}{a}\right)\right]dx,$$

$$I_2=\int_0^b G_i^2(x)dx=\int_0^a\left[\left(\cosh\beta_i\cdot\frac{x}{a}-\cos\beta_i\frac{x}{a}\right)-k_i\left(\sinh\beta_i\frac{x}{a}-\sin\beta_i\frac{x}{a}\right)\right]^2dx,$$

$$I_3=\int_0^b F_j^2(y)dy=\int_0^b\left[\left(\sinh\beta_j\frac{y}{b}+\sin\beta_j\frac{y}{b}\right)-k_j\left(\sinh\beta_j\frac{y}{b}-\sin\beta_j\frac{y}{b}\right)\right]^2dy,$$

$$I_4 = \left(\frac{\beta_i}{a}\right)^2 \int_0^a G_i''(x) \cdot G_i(x) dx = \left(\frac{\beta_i}{a}\right)^2 \int_0^a \left[\left(\cosh\beta_i \frac{x}{a} - \cos\beta_i \frac{x}{a}\right) - k_i \left(\sinh\beta_i \frac{x}{a} - \sin\beta_i \frac{x}{a}\right)\right]'' \cdot$$

$$\cdot \left[\left(\cosh\beta_i \frac{x}{a} - \cos\beta_i \frac{x}{a}\right) - k_i \left(\sinh\beta_i \frac{x}{a} - \sin\beta_i \frac{x}{a}\right)\right] dx,$$

$$I_5 = \left(\frac{\beta_j}{b}\right)^2 \int_0^b \left[\left(\sinh\beta_j \frac{y}{b} + \sin\beta_j \frac{x}{a}\right) - k_j \left(\sinh\beta_j \frac{y}{b} - \sin\beta_j \frac{y}{b}\right)\right]'' \cdot$$

$$\cdot \left[\left(\sinh\beta_j \frac{y}{b} + \sin\beta_j \frac{x}{a}\right) - k_j \left(\sinh\beta_j \frac{y}{b} - \sin\beta_j \frac{y}{b}\right)\right] dy,$$

$$I_6 = \left(\frac{\beta_j}{b}\right)^4 \int_0^b \left[\left(\sinh\beta_j \frac{y}{b} + \sin\beta_j \frac{x}{a}\right) - k_j \left(\sinh\beta_j \frac{y}{b} - \sin\beta_j \frac{y}{b}\right)\right]^{IV} \cdot$$

$$\cdot \left[\left(\sinh\beta_j \frac{y}{b} + \sin\beta_j \frac{x}{a}\right) - k_j \left(\sinh\beta_j \frac{y}{b} - \sin\beta_j \frac{y}{b}\right)\right] dy \cdot$$

By entering the integrals in the parameter expression (4) are obtained their values.

4. Results and Discussions

For the square plate corresponding to the normal modes of vibration (1,1), (2,1), (3,1) is obtained pulsation parameter values, shown in table 1.

Pulsations Parameters Table 1

Modes	(1,1)	(2,1)	(3,1)
$\sqrt{\lambda_{ij}}$	22,37	61,6	102,9

The values of the natural vibration shapes functions are shown in tables 2-4, and their shapes corresponding to the normal modes of vibration plate treated in figures3-5

Shapes functions for mode (1,1) Table 2

		Vibration Functions							Mod (1,1)		
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,032754	0,065507	0,098261	0,131015	0,163769	0,196522	0,229276	0,26203	0,294783	0,327537
0,8	0	0,107282	0,214564	0,321846	0,429128	0,536409	0,643691	0,750973	0,858255	0,965537	1,072819
1,2	0	0,189833	0,379666	0,569498	0,759331	0,949164	1,138997	1,32883	1,518662	1,708495	1,898328
1,6	0	0,252091	0,504183	0,756274	1,008366	1,260457	1,512549	1,76464	2,016731	2,268823	2,520914
2	0	0,275075	0,55015	0,825226	1,100301	1,375376	1,650451	1,925526	2,200601	2,475677	2,750752
2,4	0	0,252092	0,504183	0,756275	1,008367	1,260458	1,51255	1,764641	2,016733	2,268825	2,520916
2,8	0	0,189833	0,379666	0,5695	0,759333	0,949166	1,138999	1,328833	1,518666	1,708499	1,898332
3,2	0	0,107283	0,214565	0,321848	0,429131	0,536413	0,643696	0,750979	0,858261	0,965544	1,072827
3,6	0	0,032755	0,06551	0,098265	0,13102	0,163775	0,19653	0,229285	0,26204	0,294795	0,32755
4	0	2,14E-06	4,28E-06	6,43E-06	8,57E-06	1,07E-05	1,29E-05	1,5E-05	1,71E-05	1,93E-05	2,14E-05

Shapes functions for mode (2,1) Table 3

		Vibration Functions							Mod (2,1)		
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,144501	0,265661	0,343479	0,364099	0,321605	0,218483	0,064626	-0,12508	-0,33436	-0,5499
0,8	0	0,473301	0,870148	1,125034	1,192575	1,053389	0,715622	0,211675	-0,4097	-1,09518	-1,8014
1,2	0	0,837495	1,539706	1,990721	2,110234	1,863947	1,266276	0,374555	-0,72495	-1,93789	-3,1875
1,6	0	1,112165	2,044677	2,643609	2,802318	2,475257	1,681572	0,497396	-0,96271	-2,57345	-4,2330
2	0	1,213563	2,231094	2,884633	3,057811	2,700932	1,834884	0,542745	-1,05048	-2,80808	-4,6189
2,4	0	1,112165	2,044678	2,643611	2,80232	2,475259	1,681573	0,497396	-0,96271	-2,57345	-4,2330
2,8	0	0,837497	1,539709	1,990725	2,110239	1,863951	1,266279	0,374556	-0,72495	-1,93789	-3,1876
3,2	0	0,473304	0,870154	1,125042	1,192584	1,053396	0,715627	0,211677	-0,4097	-1,09518	-1,8014
3,6	0	0,144507	0,265671	0,343492	0,364114	0,321618	0,218492	0,064628	-0,12509	-0,33438	-0,5500
4	0	9,45E-06	1,74E-05	2,25E-05	2,38E-05	2,1E-05	1,43E-05	4,23E-06	-8,2E-06	-2,2E-05	-3,6E-05

Shapes functions for mode (3,1) Table 4

		Vibration Functions							Mode (3,1)		
y/x	0	0,4	0,8	1,2	1,6	2	2,4	2,8	3,2	3,6	4
0	0	0	0	0	0	0	0	0	0	0	0
0,4	0	0,348247	0,64024	0,827781	0,877476	0,775065	0,526542	0,155747	-0,30145	-0,80581	-1,32546
0,8	0	0,92212	1,695286	2,191874	2,323463	2,05229	1,394228	0,412402	-0,7982	-2,1337	-3,50969
1,2	0	1,150413	2,114994	2,734524	2,898691	2,560383	1,739402	0,514502	-0,99582	-2,66195	-4,37859
1,6	0	0,790552	1,453403	1,879138	1,991952	1,75947	1,1953	0,35356	-0,68432	-1,82927	-3,00892
2	0	6E-06	1,1E-05	1,43E-05	1,51E-05	1,34E-05	9,08E-06	2,68E-06	-5,2E-06	-1,4E-05	-2,3E-05
2,4	0	-0,79054	-1,45337	-1,8791	-1,99191	-1,75943	-1,19528	-0,35355	0,684301	1,82923	3,008864
2,8	0	-1,15038	-2,11494	-2,73445	-2,89861	-2,56032	-1,73936	-0,51449	0,99579	2,661881	4,378476
3,2	0	-0,92206	-1,69517	-2,19172	-2,3233	-2,05215	-1,39413	-0,41237	0,798147	2,133554	3,509441
3,6	0	-0,34811	-0,63998	-0,82745	-0,87712	-0,77475	-0,52633	-0,15568	0,301327	0,805488	1,324931
4	0	0,000307	0,000563	0,000729	0,000772	0,000682	0,000463	0,000137	-0,00027	-0,00071	-0,00117

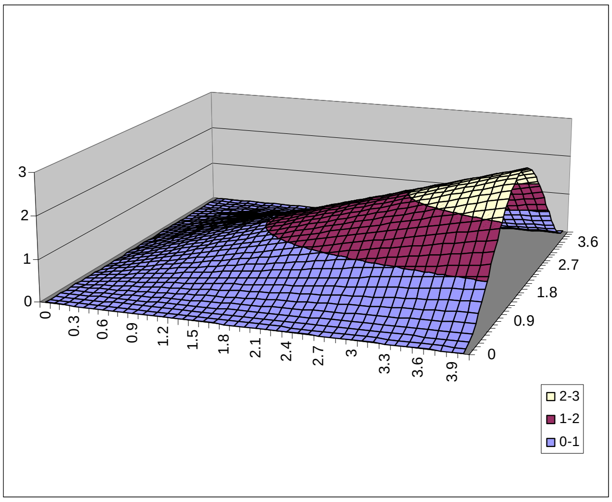

Fig. 3 Image of shape corresponding to mode (1,1)

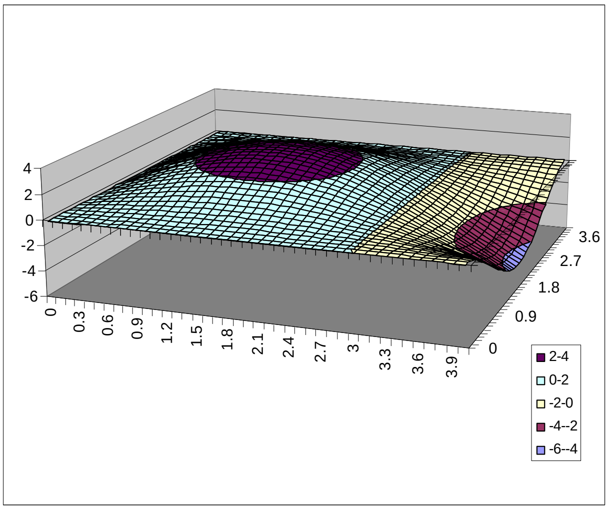

Fig.4 Image of shape corresponding to mode (2,1)

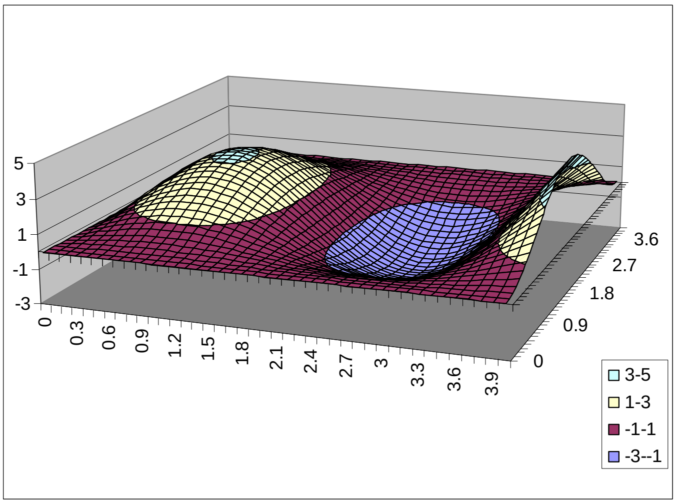

Fig. 5 Image of shape corresponding to mode (3,1)

5. Conclusions.

Solution of such problems by the classical methods is very difficult to find, or often impossible.

Vlasov's method uses linear combinations of the eigenfunctions of lateral beams vibrations, which are able to satisfy most boundary conditions.

In the case of Galerkin-Vlasov variational method adapted by the author are highlighted as follows [5]:

- The election of displacement function approximation as a linear combination between the natural vibration shapes products functions of the beams on both directions and time function which indicates that plate motion after a normal harmonic vibration which is a motion that is produce a specific pulsation;
- Calculation algorithm, which characterizes the method;
- Pulsations parameters considered for

normal modes of vibration;

- The shapes funcions for three normal modes of vibration.

In the literatureare are not published results regarding normal modes of vibration of this type of plate, the only references that can be considered are those presented by Leissa [13], which states that for the case of antisymmetric-antisymmetric vibrational mode (2.1), the parameter values are close to those determined for the case of plate clamped on two opposite sides and free on the other two. Also, an approximate value of their fundamental parameter for the square plate is given by Janich [12]. Using the Rayleigh method, Janich [12], considered shapes functions of beams vibrations as given by simple trigonometric functions of the forms:

$$\phi(x,y)=\left(\cos\frac{3\pi x}{2a}-\cos\frac{\pi x}{2a}\right)\left(1-\cos\frac{\pi y}{2b}\right) \quad (5)$$

By applying the Rayleigh method, Janich, obtained for the fundamental pulse parameter the value [12]

$$\sqrt{\lambda_{11}}=24{,}64$$

Considering the approximate percentage deviation of the parameter values determined by the method proposed, the fundamental parameter, compared to parameter value determined by Janich [12], is 9.22%. The parameter values determined for square plate, by the proposed method, and the fundamental parameter value obtained by Janich [12] are presented in Table 6.

Content of the paper has been designed so as to emphasize the essential theoretical aspects with the subtleties of physico-mathematical and practical problems of dynamic analysis of rectangular flat plates.

The work proposed by the author is intended to be an attempt to validate the Vlasov-Galerkin variational method for flat plate considered. Variational analysis performed to determine the natural vibration forms function values was done using an Excel program and for determining the parameter values using Matlab software.

The paper includes not only information from the Romanian and international literature of teachers and researchers dynamics schools, but also the values determined by the author on the characteristic dynamic determined.

Table 6

Parameters	$\sqrt{\lambda_{11}}$	$\sqrt{\lambda_{21}}$	$\sqrt{\lambda_{31}}$
Author	22,37	61,67	102,9
Janich	24,64	-	-

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