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CONTROL OF ACTIVE VEHICLE SUSPENSION USING DUAL OBJECTIV ANALYSIS

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Abstract: *When designing vehicle suspensions, the dual objective is to minimize the vertical forces transmitted to the passengers (i.e., to minimize vertical car body acceleration) for passenger comfort, and to maximize the tire-to-road contact for handling and safety. While traditional passive suspensions can negotiate this tradeoff effectively, active suspension systems have the potential to improve both ride quality and handling performance, with the important secondary benefits of better braking and cornering because of reduced weight transfer. This improvement, of course, is conditional upon the use of feedback to control the hydraulic actuators. In this paper we propose two methods for analyzing this tradeoff. First, we use frequency domain analysis of passive quarter- car suspension system and second we design a backstepping controller for analyze a parallel active suspension in witch the hydraulic actuator force is viewed as the control input.*
Keywords: *active suspension, control, transfer functions, backstepping controller*

1. INTRODUCTION

Development of control methods for passive and active suspension systems is a major topic of automotive industries. In general, ride comfort, road handling, and stability are the most important factors in evaluating suspension performance. Ride comfort is proportional to the absolute acceleration of the vehicle body, while road handling is linked to the relative displacement between vehicle body and the tires.
On the other hand, stability of vehicles is related to the tire-ground contact. The main concern in suspension design and control is the fact that currently, achieving improvement in these three objectives poses a challenge because these objectives will likely conflict with each other in the vehicle operating domain [1], [4].
A good suspension system shall improve ride quality and passenger comfort simultaneously. For ride quality improvement vertical acceleration that caused by road profile shall be limited. This means that suspension system shall absorb road disturbances. In the other word, contact of tire with road surface shall decrease. In the other side, for increasing the controllability of vehicle, tire shall contact to road more. Therefore, reach to a suitable suspension system is difficult, Because a tradeoff between ride quality and vehicle controllability exists.
In this paper we present two methods to analyze the tradeoff between ride quality and suspension travel of automotive suspensions: modal analysis to obtain approximate transfer functions and backstepping methodology to design control force generated by the actuator.

2. DYNAMICAL MODEL OF THE SUSPENSION SYSTEM

Since many of the proposed electronic suspension being considered today are independent, i.e. using local sensor information and control law, the completely active suspension system of a quarter car model, with two degrees of freedom, show in Fig. 1, has been considered in this paper. We used the following notation: m_{us} is the equivalent unsprung mass consisting of the wheel and its moving parts; m_s is the sprung mass, i.e., the part of the whole body mass and the load mass pertaining to only one wheel; k_t is the elastic constant of the tire, whose damping characteristics have been neglected.
If assume that the tire does not leave the ground the liniarized equations of the motions are

$$M\,\ddot{\boldsymbol{x}} + C\,\dot{\boldsymbol{x}} + K\,\boldsymbol{x} = D_1\,x_r + D_2\,u \tag{1}$$

$$\begin{bmatrix} m_s & 0 \\ 0 & m_u \end{bmatrix}\begin{Bmatrix} \ddot{x}_s \\ \ddot{x}_u \end{Bmatrix}+\begin{bmatrix} c_s & -c_s \\ -c_s & c_s \end{bmatrix}\begin{Bmatrix} \dot{x}_s \\ \dot{x}_u \end{Bmatrix}+\begin{bmatrix} k_s & -k_s \\ -k_s & k_s+k_t \end{bmatrix}\begin{Bmatrix} x_s \\ x_u \end{Bmatrix}=\begin{Bmatrix} 0 \\ k_t \end{Bmatrix}x_0+\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}u \tag{2}$$

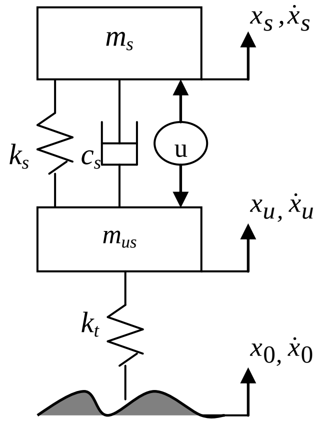

Figure 1: Quarter car model

Using this the state variables
$x_1 = x_s - x_u$, - suspension deflection (rattle space);
$x_2 = \dot{x}_s$, - the vertical absolute velocity of the sprung mass m_s;
$x_3 = x_u - x_0$, - tire deflection;
$x_4 = \dot{x}_u$, - the vertical absolute velocity of the unsprung mass m_u;
$u(t)$ - the control force produced by the actuator;
$x_0(t)$ - represents the disturbance, it coincides with the absolute vertical velocity of the point of contact of the tire with the road;
we can rewrite (1) in state space as

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{u} + L\dot{x}_0 \tag{3}$$

where

$$\boldsymbol{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix};\ A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 0 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\frac{k_t}{m_u} & -\frac{c_s}{m_u} \end{bmatrix};\ B = \begin{Bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{Bmatrix};\ L = \begin{Bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{Bmatrix} \tag{4}$$

3. PASSIVE SUSPENSION

In the case of passive suspension the control force $u(t)$ is set to zero. Since the system obtained from (2) is linear, we can use frequency domain analysis. For judge the effectiveness of the suspension system we are looking at the location of system poles and zeros, and the response of the vehicle outputs to road disturbances. System zeros can be obtained by the transfer function for the control input to the outputs.
The primary concern is acceleration transfer function:

$$H_A(s) = \frac{\ddot{x}_s(s)}{\dot{x}_0(s)} = \frac{k_t s(c_s s + k_s)}{d(s)} \tag{5}$$

where $d(s)$ is characteristic polynom

$$d(s) = m_u m_s s^4 + (m_u + m_s)c_s s^3 + ((m_u + m_s)k_s + m_s k_t)s^2 + c_s k_t s + k_s k_t \tag{6}$$

Similarly, we define the following two transfer functions: rattle space transfer function (suspension deflections) [4]

$$H_{RS}(s)=\frac{x_s(s)-x_u(s)}{\dot{x}_0(s)}=-\frac{k_t m_s s}{d(s)} \tag{7}$$

and tire deflection transfer function

$$H_{TD}(s)=\frac{x_u(s)-x_r(s)}{\dot{x}_0(s)}=-\frac{m_u m_s s^3+(m_u+m_s)c_s s^2+(m_u+m_s)k_s s}{d(s)} \tag{8}$$

We are defined the transfer functions with respect to the road input velocity ($\dot{x}_0(s)$), so all frequencies contribute equally to their mean square values. The system is observable for all three outputs and all states are controllable. This transfer functions will be used for comparison purposes later on. In this paper the numerical simulations were made for these values of parameters:

$$\begin{aligned}
&m_s=240\,Kg, \quad m_u=36\,Kg,\\
&c_s=1000\,N\cdot sec/\,m,\; c_t=0\\
&k_s=16000\,N/m,\; k_t=160000\,N/m.
\end{aligned} \tag{9}$$

4. ANALYSIS OF PASSIVE SUSPENSION USING APPROXIMATE TRANSFER FUNCTIONS

Modal decoupling will be used to study the influence of different suspension parameters on the properties of the automotive suspension.

In order to study the effects of specific suspension parameters on the suspension performance, we calculate the natural frequencies and mode shapes of the suspension system and then transform to a new set of coordinates in which the two equations of motion are approximately decoupled [4]. For the numerical parameters (9), the two approximate decoupled equations are:

- sprung mass mode approximation

$$m_s\ddot{x}_s+c_s\dot{x}_s+k_s x_s=c_s\dot{x}_0+k_s x_0,\ \text{for}\ \ |x_s|>>|x_u| \tag{10}$$

- unsprung mass mode approximation

$$m_u\ddot{x}_s+c_s\dot{x}_u+k_t x_u=k_t x_0,\ \text{for}\ |x_u|>>|x_t|\,. \tag{11}$$

So, we obtain the following approximate transfer functions:

- acceleration transfer function

$$\frac{1}{s}H_A(s)\approx\frac{x_s}{x_0}=\frac{c_s s+k_s}{m_s s^2+c_s s+k_s} \tag{12}$$

- rattle space transfer function

$$sH_{Rs}(s)\approx\frac{x_s-x_0}{x_0}=-\frac{m_s s^2}{m_s s^2+c_s s+k_s} \tag{13}$$

- tire deflection transfer function

$$H_{TD}(s)\approx\frac{x_u-x_0}{x_0}\approx\frac{-m_u s^2}{m_u s^2+c_s s+k_t} \tag{14}$$

To evaluate the acuracy of the approximate transfer functions of equations (12) and (14), Figures 2 and 3 show a comparison between the original and approximate transfer functions.
It is clear that the approximate transfer function (12) matches the original transfer function (6) well for frequency range $\omega < \omega_1$ and the approximate transfer function (14) matches the original transfer function $H_{TD}(s) = (x_u - x_0) / \dot{x}_0$ well for the frequency range $\omega > 0.5\omega_2$

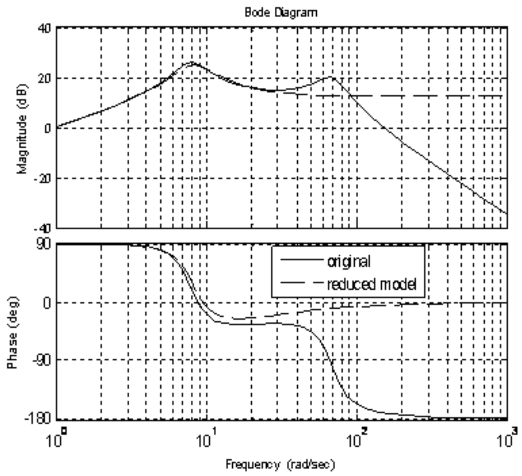

Figure 2: Bode for $H_A(s)$ sprung mass approximate mode

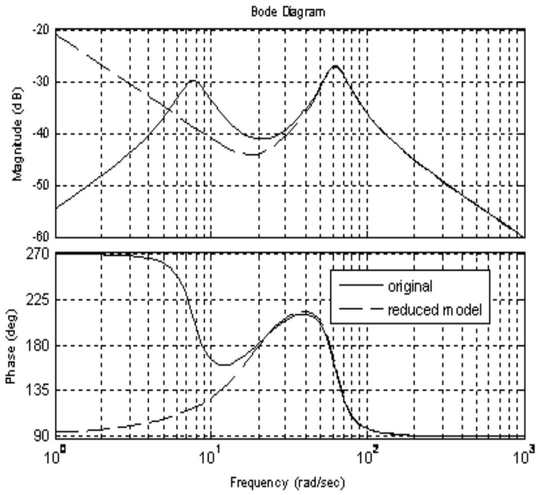

Figure 3: Bode for $H_{TD}(s)$ unsprung mass approximate mode

In order to improve passenger comfort the transfer function $H_A(s)$ from the road disturbance to the car body acceleration should be small in the frequency range from 0–65 rad/s. At the same time it is necessary to ensure that the transfer function $H_{RS}(s)$ from the road disturbance to the suspension deflection is small enough to ensure that even very rough road profiles do not cause the deflection limits to be reached.
The fact that the actuator force u is applied between the two masses places fundamental limitations on the transfer functions $H_A(s)$ and $H_{RS}(s)$. As shown in [2], [4] and [5] the acceleration transfer function has a zero at the "tyrehop frequency," $\omega_1 = \sqrt{k_t / m_u}$ .For the parameter values listed in (9), $\omega_1 = 60{,}7$ rad/s. Similarly, the suspension deflection transfer function has a zero at the "rattle space frequency," $\omega_1 = \sqrt{k_t / (m_u + m_s)}$. For the parameter values listed in (9), $\omega_2 = 24{,}07$ rad/s. The tradeoff between passenger comfort and suspension deflection is captured by the fact that is not possible to simultaneously keep both the above transfer functions small around the tyrehop frequency and in the low-frequency range.

5. ANALYSIS OF ACTIVE SUSPENSION USING BACKSTEPPING METHOD

Active suspension systems add actuators to the passive components. Active suspension systems have the potential to improve both ride quality and handling performance. This improvement is conditional upon the use of feedback to control the hydraulic actuators. In this paper we analyze the tradeoff between ride quality and suspension travel by backstepping design methodology [3].
The first step in the design of a backstepping controller is the choice of a quantity to be regulated. The choice of this variable is crucial to the performance of the closed-loop system, and one of the goals of this paper will be to show how to exploit the design flexibility built into this choice in order to achieve the desired closed-loop behavior.
If we want to minimize the car body acceleration (i.e. our objective control are the forces transmitted to passengers) then the desired value of actuator force is

$$u = k_s x_1 + c_s (x_2 - x_4) \tag{15}$$

to yield $\dot{x}_2 = \ddot{x}_s = 0$.

Substituting this expression into (4) yields the closed-loop system

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{k_t}{m_u} & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{Bmatrix} \dot{x}_0 \tag{16}$$

Now, if we want to minimize the suspension travel, $x_s - x_u$, then the regulated variable becomes x_1.
So, we obtain the zero dynamics of the closed-loop systems, which consist of an unstable subsystem

$$\begin{cases} \dot{x}_3 = x_4 - \dot{x}_0 \\ \dot{x}_4 = -\dfrac{k_t}{m_u} x_3 \end{cases} \tag{17}$$

However, the zero dynamics are again oscillatory and hence this design is still not acceptable. We must therefore choose the regulated variable so as to avoid the oscillatory zero dynamics. One such choice is the variable

$$y_1 = x_s - \tilde{x}_u \tag{18}$$

where $\tilde{x}_u$ is a filtered version of the wheel displacement x_u

$$\tilde{x}_u = \frac{e}{s+e} x_u \tag{19}$$

This choice represents the first step towards the design, backstepping design of a controller which will accommodate the inherent tradeoff between ride quality and rattle space usage. The choice of the positive constant e affects the properties of our active suspension [3].
For small values of e, (19) is a low-pass filter. Hence, the regulated variable y_1 is essentially equal to the car body displacement x_u as long as the road input contains only high-frequency components which are rejected; however, at very low frequencies (constant or slowly changing road elevations) and in steady state, y_1 becomes almost identical to the suspension travel $x_s - x_u$. Thus, as we will see later on, the sustained oscillations are eliminated, and the active suspension rejects only high-frequency road disturbances, namely the ones which generate large vertical accelerations and cause passenger discomfort.
As the value of e becomes larger, more high-frequency components of the road input are allowed to pass through the filter (19). Hence, the regulated variable $y_1$approximates the suspension travel $x_s - x_u$: the high filter bandwidth renders $\tilde{x}_u \approx x_u$. As a result, the active suspension becomes stiffer and reduces its rattlespace use, at the price of significantly reduced passenger comfort [3]. With this choice of variable y_1, defined in (18) and using backstepping techniques we get a new control law for the calculation of transfer functions relating the road input x_0 to the car body displacement x_s and wheel travel x_u.
So, for the closed-loop systems, we obtain the transfer functions

$$H_{1A} = \frac{X_s(s)}{X_0(s)} = \frac{k_t e}{d_1(s)} \tag{20}$$

$$H_{1TD}(s) = \frac{X_u(s)}{X_0(s)} = \frac{s+e}{d_1(s)} \tag{21}$$

where

$$d_1(s) = m_u s^3 + e(m_s + m_u)s^2 + k_t s + k_t e \tag{22}$$

Furthermore we compare the transfer functions (5)-(8), of the passive suspension and the transfer functions (20)-(21) of active suspension.

These transfer functions are plotted in Figs.4-6 for the numerical parameters given by relations (9). As shown in Figs. 4 -6, the frequency response plots for any real suspension must pass through certain invariant point. These invariant properties are a result of the fact that the suspension forces are applied only between a wheel and the car body, and they place insurmountable limitations to what can be achieved by active suspension designs. With small e(e=2), the active suspension design reduces both car body displacement and acceleration compared to the passive one, Figs. 4 and 5, but increases the suspension travel as seen in Fig. 6. On the other hand, if e is increased to 9, then the suspension travel can be significantly reduced, as seen in Fig. 5, but then the car body displacement and acceleration are increased.

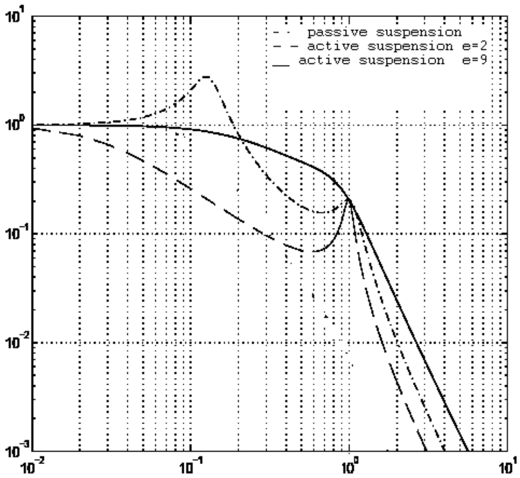

Figure 4: Car body displacement

Frequency response plot of $\left|\dfrac{X_s(j\omega)}{X_o(j\omega)}\right|$

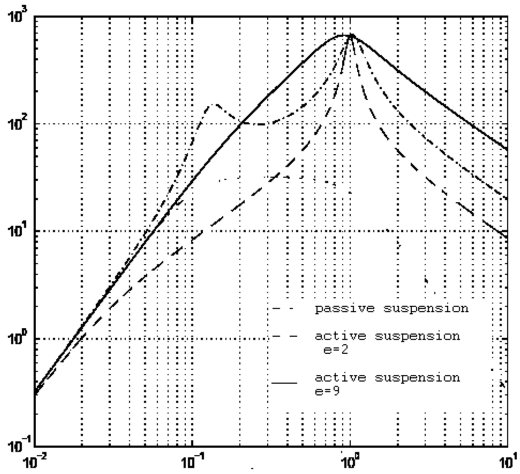

Figure 5: Car body acceleration

Frequency response plot of $\left|\dfrac{-\omega^2 X_s(j\omega)}{X_o(j\omega)}\right|$

6. CONCLUSION

Using the approximately decoupled models and backstepping method the following conclusions on suspension design were obtained:

- Decreasing suspension stiffness improves ride quality and road holding. However, it increases rattle space requirements.
- Increased suspension damping reduces resonant vibrations at the sprung mass frequency. However, it also results in increased high frequency harshness.
- Increased tire stiffness provides better road holding but leads to harsher ride at frequencies above the unsprung mass frequency.
- It was shown that considering ride quality and road holding trade-offs that both can be improved at low frequencies and at the sprung natural frequency.

Nevertheless, it is also clear that with the appropriate choice of the filter bandwidth e our active suspension design is superior not only to the passive suspension but also to the ideal one in some frequency ranges.

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