



# VIRTUAL PROTOTYPING OF MECHANICAL SYSTEMS USING MULTIBODY SOFTWARE

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## **1. INTRODUCTION**

In mechanical engineering, the concept of transferring of the drawings in the computer was introduced two decades ago. At the same time, major improvements were developed, enabling engineers to move from creating simple, two-dimensional drawings, to modeling three-dimensional solids. In the last years, on the basis of advanced analysis and simulation (prototyping) software, the designers have the possibility to build models of not just parts but entire mechanical systems, and then to simulate their behavior and optimize the design long before building a physical prototype.

Determining the real behavior is a priority in the dynamic analysis of the mechanical systems since the emergence of the computer graphic simulation. Recent publications reveal a growing interest on analysis methods for multi-body systems that may facilitate the self-formulating algorithms, having as main goal the reducing of the processing time in order to make possible real-time simulation [2, 3, 5, 6, 7, 10]. These methods were used to develop powerful modeling and simulation environments, namely MBS (Multi-Body Systems) programs, which allow building and simulating a computer model of any mechanical system that has moving parts [11, 13].

The mechanical systems analysis and simulation software automatically formulate and solve the dynamic equations of motion taking into consideration the geometric - elastic model of the mechanical system and the constraints in motion (geometric and kinematic constraints). These type of programs were lanced in commercial versions even in the 70's but in the last decade a new type of studies were defined through their use: Virtual Prototyping. This technology consists mainly in conceiving a detailed model and using it in a virtual experiment, in a similar way with the real case. Virtual Prototyping is a software-based engineering process that enables modeling the mechanical system, simulating its motion under real operating conditions and, finally, optimizing the system.

An important advantage of this kind of analysis / simulation consists in the possibility of make virtual measurements in any point and area of the mechanical system and for any parameter (displacements, velocities, accelerations, forces etc.). Thus, the designers can make quick decisions on any design changes without going through expensive physical prototype building and testing.

#### 2. MECHANICAL SYSTEMS ANALYSIS AND SIMULATION SOFTWARE

The virtual prototyping platform includes three types of computer programs:

- CAD (Computer Aided Design) software, for example CATIA, PROENGINEER, EUCLID, AUTOCAD;
- MBS (Multi Body Systems) software, for example ADAMS, DYMES, SD-EXACT, PLEXUS;
- FEA (Finite Element Analysis) software, for example NASTRAN, PATRAN, NISA, COSMOS, ANSYS.

The CAD software is used to creating the geometric model of the mechanical system (i.e. solid modeling). The solid model contains information about the mass and the inertia properties of the bodies (rigid parts) that form the mechanical system. At the same time, the CAD environment provides the ability to perform simple motion studies and to easily transfer geometry between CAD system and virtual prototype software. The part's geometry can be exported from CAD environment to MBS environment by using standard format file, for instant an IGES (Initial Graphics Exchange Standard) file. To import the geometry of the body, the MBS software reads the CAD file and converts the geometry into a set of MBS geometric elements.

The MBS software (namely virtual prototyping software), which is the main component of the virtual prototyping platform, allows to analyze and simulate (animate) the mechanical system. The major difference of mechanical system dynamics from the conventional structural system dynamics is the presence of a high degree of geometric non-linearity associated with large rotational kinematics. Governing equations for conventional structural system dynamics are linear differential equations, while those equations for mechanical system dynamics are nonlinear differential equations that are coupled with nonlinear algebraic equations of cinematic constraints.

The FEA software is used to modeling flexible bodies in mechanical systems. It provides the ability to transfer loads from virtual prototyping to FEA and to bring component flexibility from FEA back into virtual prototyping. Integrating flexible body into model allows to capture inertial and compliance effects during handling and comfort simulations, study deformations of the flexible components, and predict loads with greater accuracy, therefore achieving more realistic results. The flexible body characteristics are defined in a finite element modeling (FEM) output file that are usually called modal neutral file (MNF). The information in an MNF includes: geometry (location of nodes and node connectivity), nodal mass and inertia, mode shapes, generalized mass and stiffness for modal shapes

The general scheme of the virtual prototyping platform, which describes the steps to creating the first physical prototype beginning with the virtual prototype, is presented in figure 1. The connections between the virtual prototyping platform's components define a factor that is, usually, called "Integration" [13]. The steps to create a virtual model of mechanical system with the MBS software mirror the same steps to build a physical prototype (see figure 1).

The mechanical system is characterized as a constrained, multi-body, spatial mechanical system, in which rigid bodies (parts) are connected through geometric constraints (joints), cinematic constraints (motion generators), compliant joints and force elements such as springs and dampers.

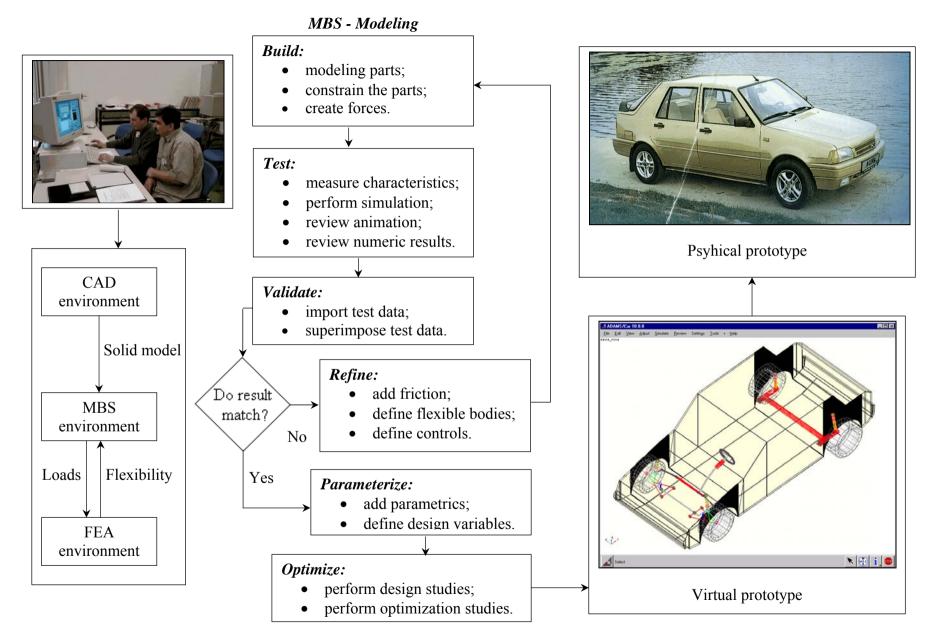


Fig. 1. Virtual Prototyping Platform

For a mechanical system, the type of analysis that can be performed depends on the degrees of freedom (DOF) of the model. Degrees of freedom are a measure of how parts can move relative to one another in a model. Each degree of freedom corresponds to at least one equation of motion. Constraints define how bodies are attached and how they are allowed to move relative to each other. Each constraint removes a number of degrees of freedom. The total number of degrees of freedom in the model is equal to the difference between the number of allowed motions and the number of active constraints (geometric and cinematic constraints):  $DOF=6\cdot n-r$  (Gruebler court).

The analysis flow chart of the multi-body environments is shown in figure 2 [11]. There are seven different analysis options available in MBS software for a mechanical system. These analyses can be performed in separate executions, or together in a certain sequence depending on DOF.

Assembly (position) analysis allows assembling all the parts in a system at joints where the parts are connected together. Input to the analysis is a set of measured positions and orientations of all parts from design draft of the mechanical system. Output from the analysis is a set of those values that minimizes constraint errors.

Redundancy analysis is to eliminate redundant constraints from an over-constrained system. Input to the redundancy analysis is the assembled configuration of the system. Output is a remodeled system without redundant constraints.

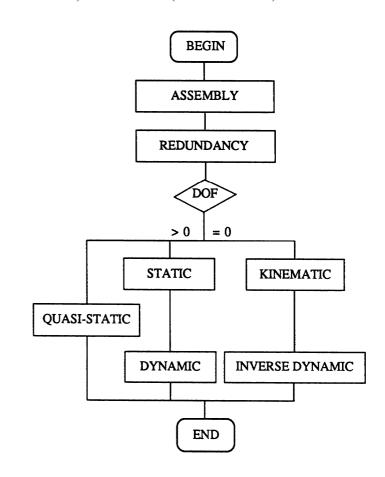


Fig. 2. MBS analysis flow chart

In the real (physical) mechanical system, it might be necessary to have different constraints that restrict the same degree of freedom because of deformation of the parts and joint-play in the connections. In the multi-body systems (mathematical) model, because the parts are rigid and joints do not permit any play, only one constraint is required and the others are redundant. Redundant constraints can be consistent or inconsistent. A redundant constraint is consistent if a solution satisfying the set of independent constraint equations also satisfies the set of dependent or redundant constraint equations. The MBS programs remove the redundant constraints from the mechanical system, consequently from the set of equations, and provide a set of results that define the motion and forces in the model.

For instant, the mechanical system shown in figure 3.a, which is a four-bar mechanism with three mobile parts and four revolute joints (each revolute joint removes 5 DOF), has the degree of freedom:

DOF =  $6 \cdot n - r = 6 \cdot 3 - (5 + 5 + 5) = -2$ .

Therefore, the mechanism contains 3 redundant constraints. These correspond to the motions that aren't allowed due to the mode in which the parts are connected in mechanism. The redundancies can be removed by changing two revolute joints (for example, A and C) to a spherical joint (A), respectively a cylindrical joint (C). In this way, the unconstrained model (fig. 3.b) has one DOF:

DOF =  $6 \cdot n - r = 6 \cdot 3 - (3 + 5 + 4 + 5) = 1$ .

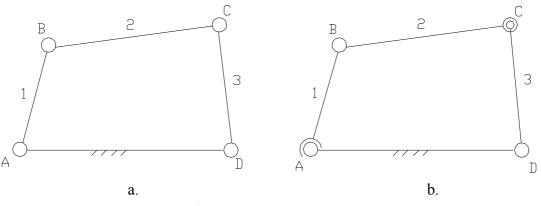


Fig. 3. Example of mechanical system with redundancies

Static analysis is to find a stable equilibrium configuration with zero velocity and acceleration of the mechanical system. Input to analysis are positions, orientations of all parts from the mechanical system with the forces acting on them. Output from the static analysis is the static equilibrium configuration (the positions and the orientations of the parts), and the reactions forces in the mechanical system.

Quasi-static analysis is a series of static equilibrium analyses for different loading conditions of forces or constraint values.

Kinematic analysis is to calculate time history of motion without considering forces and mass effects on the motion. Input to kinematics are the assembled configuration of the mechanical system and time dependent driving constraints. Outputs from the analysis are time histories of positions, velocity and acceleration of the mechanical system.

Dynamic analysis is to calculate time history of the mechanical system motion due to forces acting on the system. Inputs to the dynamics are external and internal forces, and the assembled configuration of the mechanical system. Outputs from the dynamics are time histories of positions, velocities, accelerations of the parts and the reaction forces.

Inverse dynamic analysis is to determine the constraint forces that are required to generate the prescribed motion of a kinematic system. Input to the analysis is a zero DOF system with mass properties and forces elements defined. Output from the inverse dynamic analysis is the same as in the dynamic analysis.

#### **3. DYNAMIC EQUATIONS OF MOTION**

The current position of each part that forms the mechanical system is defined by six generalized coordinates, representing the part/local reference system (LCS) position and orientation relative to the global coordinate system (GCS), also called global reference frame, that is an inertial frame attached to the ground. For a mechanical system with "n" mobile parts, the total number of generalized coordinate will be " $6 \cdot n$ ", but not all of them are independent because of the geometric and cinematic constraints. The number of degree of freedom of the mechanical system, which can be calculated with Grubler count (see chapter 2), define the number of independent generalized coordinates. The generalized coordinates are given for the initial position of the mechanical system, their evolution during the motion being governed by a set of constraint cinematic equations.

Therefore, the motion equations include the geometric constraint equations and a set of DOF differential equations, corresponding to the independent generalized coordinates, which can be formulated with different formalisms (ex. Lagrange or Newton - Euler formulation). For instant, according to the Newton - Euler formulation [5], we have:

$$\begin{bmatrix} [\mathbf{m}_{i}][\mathbf{0}]\\ [\mathbf{0}][\mathbf{I}_{i}] \end{bmatrix} \cdot \begin{bmatrix} [\ddot{\mathbf{r}}_{i}]\\ [\ddot{\mathbf{\theta}}_{i}] \end{bmatrix} = \begin{bmatrix} [\mathbf{R}_{i}]\\ [\mathbf{M}_{ii}] + [\widetilde{\boldsymbol{\omega}}_{i}] \cdot [\mathbf{I}_{i}] \cdot [\boldsymbol{\omega}_{i}] \end{bmatrix}_{i=1..7},$$
(1)

where:  $[m_i]$  - mass diagonal matrix of part 'i',

$$\begin{bmatrix} m_{i} \end{bmatrix} = \begin{bmatrix} m_{i} & 0 & 0 \\ 0 & m_{i} & 0 \\ 0 & 0 & m_{i} \end{bmatrix};$$

[I<sub>i</sub>] - inertia tensor matrix of part 'i,:

$$[I_{i}] = \begin{bmatrix} I_{xx} - I_{xy} - I_{xz} \\ - I_{xy} I_{yy} - I_{yz} \\ - I_{xz} - I_{yz} I_{zz} \end{bmatrix};$$

 $[\ddot{\theta}_i]$ ,  $[\ddot{r}_i]$ - linear / angular generalized accelerations of part 'i', which can be determined by differentiating the restriction equations and then the velocities system in respect to time;  $[\omega_i]$  - angular velocity column matrix of part 'i',

$$\left[\boldsymbol{\omega}_{i}\right] = \begin{bmatrix} \boldsymbol{\omega}_{X} \\ \boldsymbol{\omega}_{Y} \\ \boldsymbol{\omega}_{Z} \end{bmatrix};$$

 $[\widetilde{\omega}_{i}]$  -  $[3 \times 3]$  antisymmetrical matrix,

$$\begin{bmatrix} \widetilde{\boldsymbol{\omega}}_{i} \end{bmatrix} = \begin{bmatrix} \mathbf{0} - \boldsymbol{\omega}_{z} & \boldsymbol{\omega}_{Y} \\ \boldsymbol{\omega}_{z} & \mathbf{0} - \boldsymbol{\omega}_{X} \\ - \boldsymbol{\omega}_{Y} & \boldsymbol{\omega}_{X} & \mathbf{0} \end{bmatrix};$$

 $[R_i]$ ,  $[M_i]$  - resultant force / torque acting on part 'i', which can be determined from the equilibrium equations of the part.

As example, we'll considerate the mechanical system shown in figure 4, which is used for the guidance of the rear axle of the passenger cars. The model contains five mobile parts: car body (1), rear axle (2), lower guiding links (3, 4) and upper guiding arm (5). The constraints, which define how parts are attached and how they are allowed to move relative to each other, are represented by idealized joints (geometric constraints) and motions generators (cinematic constraints) that drive the model. The connections of the upper and lower arms to car body and axle were modeled as spherical joints, which allows the free rotation about a common point of one part with respect to another part. Joining the triangular upper link to car body by two spherical joints, a revolute joint is obtained.

In the lack of the front suspension, modeling a fictive joint between car body and ground ensures the car body equilibrium. The car body equilibrium can be made with a spherical joint placed in the longitudinal plan of vehicle. The location ( $C_0$ ) of the spherical joint was obtained on the basis of double conjugate point's theory [1].

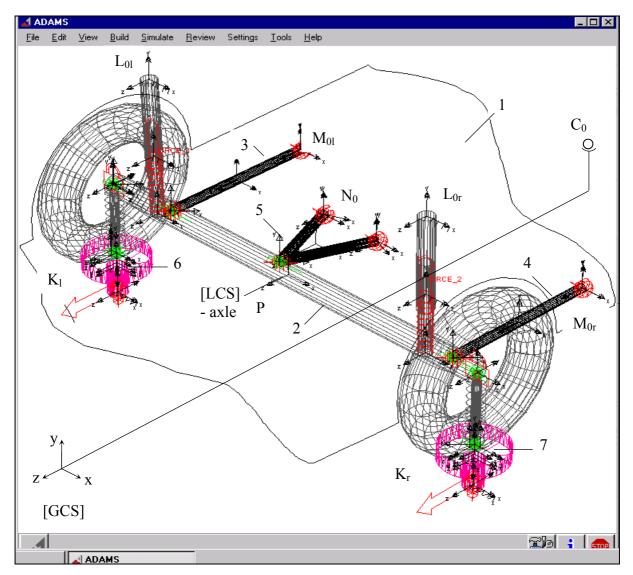


Fig. 4. The MBS dynamic model of a rear axle suspension

To modeling the drivers that dictate the movement of a part as a function of time, general point motion is used. This prescribes the motion of two parts along or around the three axes. In paper, the suspesion system is analyzed in passing over bumps regime, therefore two additional parts (6, 7), with negligible masses and inertia properties, are used to model the tire contact patches. The roadway profile is modeled by driver constraints, which are applied to tire patches (left and right) and to move wheels.

Elastic and damping elements of the suspension system represents forces acting between two parts (car body and axle, respectively) over a distance and along a particular direction. The suspension spring is modeled as a double active (tension – compression) elastic element of translational nature, between car body and axle. The inputs for spring are: global coordinates of the points in which the springs are connected to adjacent parts; undeformed spring length; constant spring stiffness or spring force vs. deflection characteristic. The internal forces of elastic bumpers have transitory character, so that these elastic elements was modeled as translational springs with unilateral rigidity, which are active only when spring is in tension or in compression, using an one-sided impact force. The tire was modeled as a three-dimensional Hertz model that contains a spring in parallel with a damper, one for each direction, between axle (rims) and ground..

Degree of freedom of the model, which is equal to the difference between the number of allowed part motions and the number of active constraints, will be:

- generalized coordinates for 7 mobile parts (car body, axle, lower/upper links, cylinders tire patches): 7 × 6 = 42;
- degrees of freedom restricted by constraints:
  - spherical joint between car body and ground:  $1 \times (-3) = -3$ ,
  - spherical joints between guiding links and axle / car body:  $5 \times (-3) = -15$ ,
  - revolute joint between upper arm and car body:  $1 \times (-5) = -5$ ,
  - translational joints between tire patches and ground:  $2 \times (-5) = -10$ ,
  - motions generators (drivers):  $2 \times (-1) = -2$ ;

DOF = 42 - 35 = 7.

Consequently, the dynamic model has 7 independent generalized coordinates, namely: three rotations ( $\varphi_{1x}$ ,  $\varphi_{1y}$ ,  $\varphi_{1z}$ ) for car body, the vertical position ( $Y_P$ ) and the roll angle ( $\varphi_{2z}$ ) for axle, the proper rotations ( $\varphi_{3z}$ ,  $\varphi_{4z}$ ) for lower links. The generalized coordinates are given for the initial position, their evolution during the simulation being governed by a set of constraint equations, as follows:

• spherical joints ( $M_l$ ,  $M_r$ , N) between guiding links – 3, 4, 5 and axle – 2:

$$F_{32} = [r_P] + [M_{20}] \cdot [r_{Ml}]_2 - [r_{M0l}] - [M_{30}] \cdot [r_{Ml}]_3 = 0,$$
  
$$F_{42} = [r_P] + [M_{42}] \cdot [r_{44}]_2 - [r_{44}]_2 - [M_{42}]_2 [r_{44}]_3 = 0,$$

$$F_{42} = [r_P] + [M_{20}] \cdot [r_{Mr}]_2 - [r_{M0r}] - [M_{40}] \cdot [r_{Mr}]_4 = 0,$$
(2)

$$\mathbf{F}_{52} = [\mathbf{r}_{\mathbf{P}}] + [\mathbf{M}_{20}] \cdot [\mathbf{r}_{\mathbf{N}}]_2 - [\mathbf{r}_{\mathbf{N}0}] - [\mathbf{M}_{50}] \cdot [\mathbf{r}_{\mathbf{N}}]_5 = 0$$

where  $[M_{i0}]$  represents the  $[3\times3]$  transformation matrix of the part 'i',  $[r_P]$  – position vector of the axle LCS origin,  $[r_{M, N}]_2$  – position vectors of guiding axle points  $M_l$ ,  $M_r$ , N in axle system,  $[r_{M, N}]_{3-5}$  – position vectors of guiding points in local systems of the upper/lower links;

• spherical  $(M_{0l}, M_{0r})$  and revolute  $(N_0)$  joints between links – 3, 4, 5 and car body – 1:

$$F_{31} = \begin{bmatrix} X_{M01} \\ Y_{M01} \\ Z_{M01} \end{bmatrix} - \begin{bmatrix} X_{C0} \\ Y_{C0} \\ Z_{C0} \end{bmatrix} - \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = 0, \qquad F_{41} = \begin{bmatrix} X_{M0r} \\ Y_{M0r} \\ Z_{M0r} \end{bmatrix} - \begin{bmatrix} X_{C0} \\ Y_{C0} \\ Z_{C0} \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = 0,$$

$$F_{51} = \begin{bmatrix} X_{N0} \\ Y_{N0} \\ Z_{N0} \\ \varphi_{5y} \\ \varphi_{5z} \end{bmatrix} - \begin{bmatrix} X_{C0} \\ Y_{C0} \\ Z_{C0} \\ \varphi_{1y} \\ \varphi_{1z} \end{bmatrix} - \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix} = 0;$$
(3)

• spherical joint (C<sub>0</sub>) between car body and ground:

$$F_{10} = \begin{bmatrix} X_{C0} \\ Y_{C0} \\ Z_{C0} \end{bmatrix} - \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = 0;$$
(4)

• translational joints  $(K_l, K_r)$  between tire patches – 6, 7 and ground:

$$F_{60} = \begin{bmatrix} X_{Kl} \\ Z_{Kl} \\ \varphi_{6x} \\ \varphi_{6y} \\ \varphi_{6z} \end{bmatrix} - \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} = 0, \quad F_{70} = \begin{bmatrix} X_{Kr} \\ Z_{Kr} \\ \varphi_{7x} \\ \varphi_{7y} \\ \varphi_{7z} \end{bmatrix} - \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix} = 0.$$
(5)

The geometric constants  $a_i$ ,  $b_i$ ,  $c_j$ ,  $d_i$ ,  $e_j$ ,  $f_j$  can be obtained from the initial position of the suspension. The constraint equations form a system of 23 scalar relations between the 30 generalized coordinates. Therefore, another 7 differential equations are necessary. These equations were obtained by using the Newton – Euler formalism. In relation (1), resultant force and torque acting on part 'i' were be determined from the equilibrium equations of each part.

As example, for car body, taking into account the elastic and damping forces (applied in points  $L_{0l}$ ,  $L_{0r}$ ), the reaction forces and torques in the joints to lower/upper links ( $M_{0l}$ ,  $M_{0r}$ ,  $N_0$ ) respectively in the spherical joint to ground ( $C_0$ ), and the mass  $G_1$  of part, the resultant force/torque will be (fig. 5):

 $[\mathbf{R}_{1}]_{1} = [\mathbf{F}_{M01}]_{1} + [\mathbf{F}_{M0r}]_{1} + [\mathbf{F}_{N0}]_{1} + [\mathbf{F}_{L01}]_{1} + [\mathbf{F}_{L0r}]_{1} + [\mathbf{F}_{C0}]_{1} + [\mathbf{M}_{10}]^{\mathrm{T}} \cdot [\mathbf{G}_{1}],$ (6)

 $[M_1]_1 = [F_{M0l}]_1 \cdot [r_{M0l}]_1 + [F_{M0r}]_1 \cdot [r_{M0r}]_1 + [F_{N0}]_1 \cdot [r_{N0}] + [F_{L0l}]_1 \cdot [r_{L0l}]_1 + [F_{L0r}]_1 \cdot [r_{L0r}]_1 + [F_{L0r}]_1 + [F_{L0r}]_1 \cdot [r_{L0r}]_1 + [F_{L0r}]_1 + [F_{L0r}$ 

 $[F_{C0}]_1 \cdot [r_{C0}]_1 + [M_{N0}]_1,$ 

where:  

$$\begin{bmatrix} F_{N0} \end{bmatrix}_{I} = \begin{bmatrix} F_{N0}^{X} \\ F_{N0}^{Y} \\ F_{N0}^{Z} \end{bmatrix}_{I}, \begin{bmatrix} M_{N0} \end{bmatrix}_{I} = \begin{bmatrix} 0 \\ M_{N0}^{Y} \\ M_{N0}^{Z} \end{bmatrix}_{I}; \begin{bmatrix} F_{M0|I,r} \\ F_{M0|I,r}^{Y} \end{bmatrix}_{I} = \begin{bmatrix} F_{M0|I,r}^{X} \\ F_{M0|I,r}^{Z} \\ F_{M0|I,r}^{Z} \end{bmatrix}_{I};$$

$$\begin{bmatrix} F_{L0|I,r} \end{bmatrix}_{I} = \begin{bmatrix} F_{L0|I,r}^{X} \\ F_{L0|I,r}^{Y} \\ F_{L0|I,r}^{Z} \end{bmatrix}_{I}; \begin{bmatrix} F_{C0} \end{bmatrix}_{I} = \begin{bmatrix} F_{C0}^{X} \\ F_{C0}^{Y} \\ F_{C0}^{Z} \\ F_{C0}^{Z} \end{bmatrix}_{I}.$$

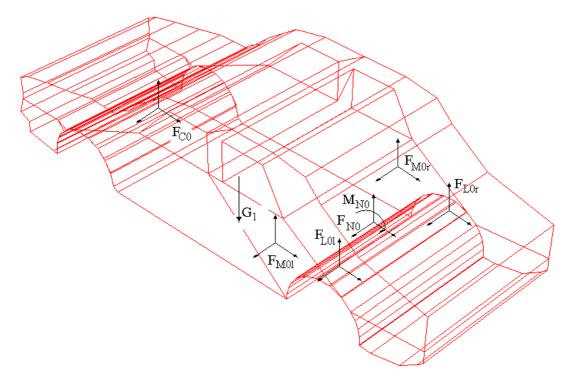


Fig. 5. Car body equilibrium

Solving the equation (1), seven differential equations will be obtained that along the constraint equations (2-5) determine the mixed system of dynamic equations.

Every major automotive manufacturer use virtual prototypes to refine and prove out their designs of suspensions, and test-drive entire vehicles in the computer, running them through a full range of maneuvers, under various driving conditions. In suspension design, the following applications can be realized: optimizing suspension design, predicting suspension characteristic, load analysis, establishing wheel envelopes, packaging studies.

#### 4. CONCLUSIONS

The multi-body environments, which automatically formulate and solve the dynamic equations of motion, allow building virtual prototypes of mechanical systems. Virtual Prototyping brings several advantages: reduce the time and cost of new product development; reduce the product cycles; reduce the number of expansive physical prototypes, and experiment with more design alternatives. The designers can quickly exploring multiple design variations, testing and refining until optimizing mechanical system behavior, long before building the first physical prototype.

Virtual Prototyping allows designers to: graphically create and assemble the parts into a system, called Virtual Prototype; run a standard set of parametric design simulations or design of experiment tests; visually compare the 3D motion performance of the design variations with sophisticated animations.

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