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STUDIES ON THE DYNAMICS OF WHEELED TRACTORS IN AGGREGATION WITH BIAXIAL TRAILERS

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Abstract: *The paper covers the dynamics of the aggregate consisting of wheeled tractors with biaxial trailers on which there are given the system of forces and moments which are reduced in the couple between the tractor and trailer.This study demonstrates the complex stress on the couples and the necessity that they should pass national legislation and international RNTR-2, as far as the degrees of mobility that the couple should allow.*
Keywords: *tractor, trailer coupling, forces, moments.*

1. THE FORCES AND MOMENTS ACTING ON A WHEELED TRACTOR WITH BIAXIAL TRAILER IN THE GENERAL CASE

Figures 1.1 and 1.2 present the aggregate consisting of tractor and trailer ascending on a slope and descending in the state of full breaking on the front and rear wheels.

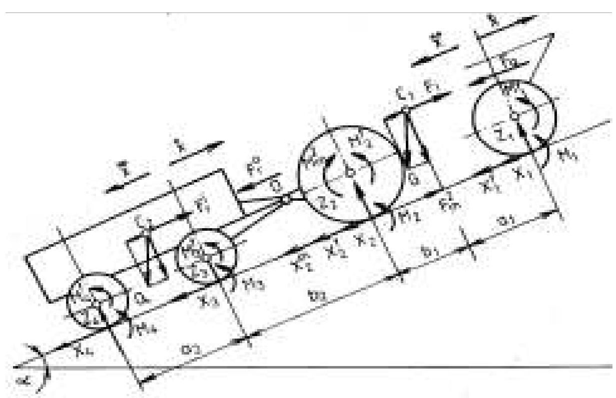


Figure 1.1

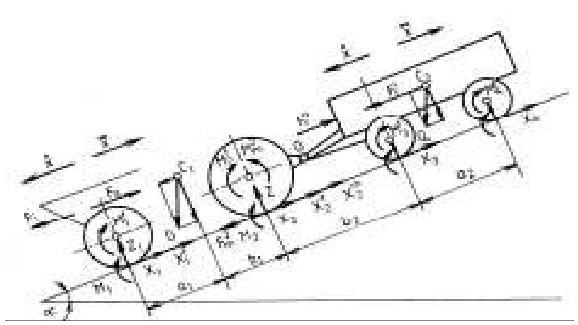


Figure 1.2

The following forces and moments act on the tractor::
G - weight of the tractor, z_1, z_2 - the reactions of the runway on the tractor wheels, M_1, M_2 – the moments of resistance to rolling (the friction and rolling moments of the tractor wheels); Fa – the global resistance force acting on the tractor, X_1, X_2 – the tangential reaction on the tractor wheels, Mn – the engine moment transmitted to the motor wheels (rear wheels); F_m^2 – the motor force developing in the process of interaction of the runway system with the soil under the action of the engine moment; G sinα – the component parallel to the inclined plane of the runway; M_{1F}, M_{2F} - the breaking moments applied to the front and rear wheels of the tractor; X_1^F, X_2^F - the total breaking forces on the front and rear wheels corresponding to the breaking moments M_1^F, M_2^F ;; X_2^m - the resistance force in the engine reduced to the engine wheels which develops in the case of breaking without changing gear; F_i - the inertia force developing during deceleration (breaking) of the tractor.
The following forces and moments act on the trailer:
Q – the trailer weight (Qsinα, Qcosα – the components respectively parallel with the inclined plane and normal to the inclined plane of the runway); Z_3, Z_4 - the reactions of the runway on the trailer wheels; M3, M4 – the global resistance force acting on the trailer; F_r^a the tangential reaction of the trailer wheels; X3, X4 - the breaking moments applied to the trailer wheels; - M_3^F, M_4^F - the inertia force. developing on breaking the trailer.If the aggregate is powered by a 4x4 tractor, then the following forces and moments should be placed on the front wheels: M_m^1 - the engine moment transmitted to the front wheels; F_m^i -the motor force developing in the process of interaction of the rolling system with the soil; X_1^m - the resistance force reduced to the front wheels in the engine which develops in the case of breaking without changing gear.

2. THE REDUCTION OF THE SYSTEM OF FORCES AND MOMENTS ACTING ON THE TRACTOR AND ON THE BIAXIAL TRAILER.

Fig. 1.3 presents the forces and moments acting on the tractor under full breaking on a slope and their mode of calculation.

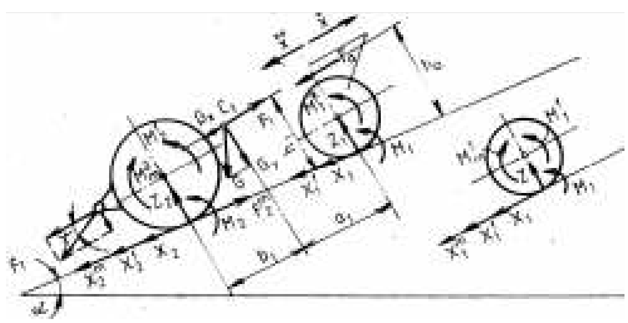


Figure 1.3

Tractor weight. For the dynamic study of the tractor the weight such as G is considered , applied to the center of weight of coordinates a1, b1 and h1, which breaks down into two components GsinαGx and GcosαGy. The static and dynamic distribution of weight on the two bridges, front and rear influence the traction qualities and stability of the tractor in aggregate with the biaxial trailer.

The resistance force to rolling.

$F_f = f(Z_1+Z_2) = X_1+X_2 = f(Gcos\alpha \pm F_t tg\gamma) = f(G_y \pm tg\gamma)$ (1)

in which: f – the coefficient of rolling resistance; F – the force of resistance of the trailerγ - the angle formed by the force Ft with the road surface.
During the displacement on the aggregate on a horizontal road, the rolling resistance force has the expression

$F_f = f(G \pm tg\gamma),$ (2)

and if the resistance force of the trailer is parallel to the road surface the expression of the force becomes:

$F_f = fG$ (3)

The resistance force due to inertia.

The forces of inertia appear in the phase of acceleration and have an opposite direction to the movement in the phase of deceleration with the same direction of the movement of the tractor and are determined by the translational and rotational kinematics. The masses in the rotational movement are unrelated/related to the kinematics of transmission. This way the resistance force due to inertia is the result of the following inertia forces:
- the forces of inertia of the tractor masses in the translation movement:

$$F_t^i = \frac{G}{g} \cdot \frac{d\dot{x}}{dt} = M\ddot{x} \tag{4}$$

:in which: M - the mass of the tractor; $\ddot{x}$ - Acceleration/deceleration of the movement of translation.
- the forces of inertia of the masses of the tractor from the rotation movement kinematically unrelated to the transmission (driven wheels):

$$M_1^i = 2J_1 \frac{d\omega_1}{dt}; \quad F_1^i = \frac{2J_1}{r_i}\dot{\omega}_1 = \frac{2J_1 d\dot{x}}{r_1^2 dt} \tag{5}$$

in which: M_1^i - the couple of inertia for the acceleration/deceleration of the driven wheels; J_1 - the moment of inertia of the driven wheels; $\omega_1, \dot{\omega}_1$ - the angular velocity respectively the angular acceleration/deceleration of the driven wheels; r_1 - the dynamic range of the driven wheels; F_i^1 - the force of inertia of the driven wheels.
- the force of inertia of the masses in the rotation movement kinematically related to the transmition (motor wheels):

$$M_2^i = 2J_2 \frac{d\omega_2}{dt}; \quad F_2^i = \frac{2J_2}{r_2}\dot{\omega}_2 = \frac{2J_2 d\dot{x}}{r_2^2 dt} \tag{6}$$

in which: M_2^i - the couple of inertia for the acceleration/deceleration of the motor wheels; J_2 - the moment of inertia of the motor wheels; F_2^i - the force of inertia of the motor wheels; $\omega_2, \dot{\omega}_2$ - the angular velocity respectively the angular acceleration/deceleration of the motor wheels, r_2 - the dynamic range of the motor wheels.
the force to the motor wheels required to defeat the moment needed for the start of the flywheel and for other rotating parts of the engine:

$$M_v^i = J_M \frac{d\omega}{dt}; \; F_v^i = \frac{J_M i_{tr} \cdot \eta_{tr}}{r_2} \cdot \frac{d\omega}{dt} = \frac{J_M i_{tr}^2 \cdot \eta_{tr}}{r_2^2} \cdot \frac{d\dot{x}}{dt} \tag{7}$$

in which: J_M - the moment of inertia of the masses of the engine being in motion; ; $d\omega / dt$ - the angular velocity of the crankshaft; i_{tr} - the total ratio of transmission of the transmission; η_{tr} - the efficiency of the transmission.
the necessary force for the defeat of inertia of a certain organ of transmission placed between the engine and the motor wheels.

$$F_x^i = \frac{J_x i_x \eta_x}{r_2} \cdot \frac{d\omega_x}{dt}; \; F_v^i = \frac{J_x i_x^2 \eta_x}{r_2^2} \cdot \frac{d\dot{x}}{dt} \tag{8}$$

in which: Jx – the moment of inertia of a certain organ; ix – the transmition ratio of the respective organ; ; η_x - the corresponding efficiency; $d\omega_x / dt$ - the angular velocity of the respective organ.
The total force of inertia of the wheeled tractor is:

$$F_i = F_t^i + F_1^i + F_2^i + F_v^i + \sum F_x^i \tag{9}$$

In the relation (a) the expression of the forces (4), (5), (6), (7) and (8) are introduced with the result

$$F_i = \frac{G}{g}\frac{d\dot{x}}{dt}\left\{1 + \frac{g}{G}\left[\frac{2J_1}{r_1^2} + \frac{1}{r_2^2}\left(J_M i_{tr}^2 \cdot \eta_{tr} + 2J_2 + \sum J_2 + \sum J_x\, i_x^2\, \eta_x\right)\right]\right\} \tag{10}$$

It is noted:

$$\delta_{rot} = 1 + \frac{1}{M}\left[\frac{2J_1}{r_1^2} + \frac{1}{r_2^2}\left(J_M i_{tr}^2 \cdot \eta_{tr} + 2J_2 + \sum J_2 + \sum J_x\, i_x^2\, \eta_x\right)\right] \tag{11}$$

which stands for the notion of coefficient of the masses of rotation with the effect on the start of the tractor.
This way the total force of inertia is calculated with the relation:

$$F_i = \pm\delta_{rot}\frac{g}{g}\frac{d\dot{x}}{dt} = \pm\delta_{rot} M \frac{d\dot{v}}{dt} = \pm\delta_{rot} M\ddot{x} \tag{12}$$

Under conditions of full breaking of the tractor, the following forces and moments should be mentioned:

breaking moments M_1^F applied to the front wheels which determine the breaking force X_1^F ;

breaking moments M_2^F applied to the rear wheels which determine the breaking force X_2^F ;

the force of resistance in the engine reduced to the motor wheels of the tractor X_2^m which is produced in the case of breaking without changing gear.

The system of forces and moments acting on the tractor with full breaking and reduced in the point zero of coupling the tractor with the trailer is synthetically presented in figure 1.4.

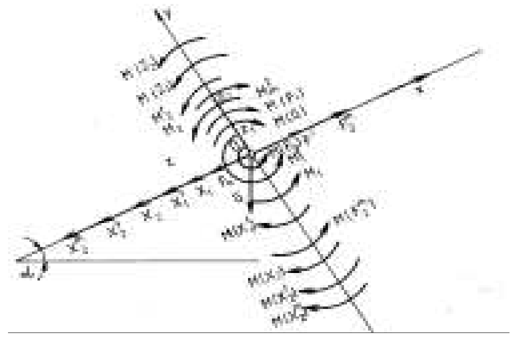


Figure 1.4

Considering the coplanar system of forces belonging to the plan xoy, there results the components of the resulting force and resulting moment, thus:

$$R_t^x = +F_i + F_2^m - F_a - X_1 - X_1^F - X_2 - X_2^F - X_2^m - G_x \tag{13}$$

$$R_t^y = Z_1 + Z_2 - G_y \tag{14}$$

$$M_t^z = M_m^2 + M_{(G)} + M_{(F_i)} + M_{(X_1^F)} + M_{(X_2)} + M_{(X_2^F)} + M_{(X_2^m)} - M_1 - M_1^F - M_{(F_a)} - M_2 - M_2^F - \\ - M_{(Z_1)} - M_{(Z_2)} \tag{15}$$

The torsor of the system of forces and moments reduced in the point 0, is

$$\tau_{0(F_i,M_i)}^t = \begin{cases} \vec{R}_t = R_t^x\vec{i} + R_t^y\vec{j}\ ; \\ \vec{M}_0^t = M_t^z\vec{k}. \end{cases} \tag{16}$$

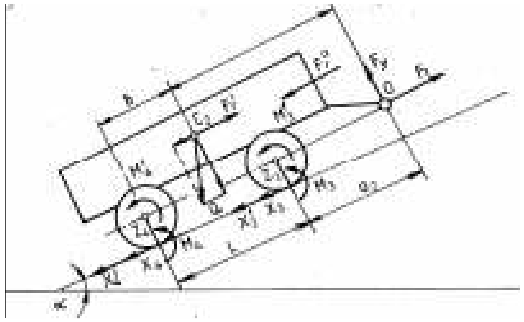
Considering the road conditions which present numerous bumps and other random disturbing factors, these determine that the system of forces should be reduced to one torsor with the elements:

$$\tau_{0(F_i,M_i)}^t = \begin{cases} \vec{R}_t = R_t^x\vec{i} + R_t^y\vec{j} + R_t^z\vec{k}\ ; \\ \vec{M}_0 = M_t^x\vec{i} + M_t^y\vec{j} + M_t^z\vec{k}. \end{cases} \tag{17}$$

whose components are variable in a discontinuos way depending on the appearance of the road profile.

Figure 1.5 presents the forces and moments under breaking conditions and the displacement on a sloping road with the purpose of establishing the reduction torsor of the system of forces at the point of coupling to the tractor, point 0.

The force of inertia F_r^i is specified which appears in the stages of acceleration/deceleration which has a strong influence on the behavior in terms of dynamics of the couple between the tractor and the trailer. The force of inertia is made up of force of inertia of the masses of the trailer in motion of translation and of the masses in the motion of rototranslation.



Figuire 1.5

The force of inertia of the trailers mass in the translation motion is:

$$F_{3t}^{i} = \frac{Q}{g}\ddot{x}\,. \qquad (18)$$

The force of inertia of the masses in the motion of rototranslation (the paralell plan motion) is:

$$M_3^i = \frac{4J_r d\omega_3}{dt} \Rightarrow F_3^i = \frac{4J_r d\omega_3}{r_3 dt}, \qquad (19)$$

where: J_r - is the moment of inertia of a wheel in ratio of the point of contact of the runway, having the expression:

$$J_r = \frac{3Qr_3^2}{2g} = \frac{3mr^2}{2}. \qquad (20)$$

The force of total inertia F_r^i of the trailer is:

$$F_r^i = \frac{Q}{g}\ddot{x} + \frac{4J_r d\omega_3}{r_3 dt} = \frac{Q}{g}\ddot{x} + \frac{4J_r\ddot{x}}{r_3^2} = \frac{Q}{g}\ddot{x}\left(1+\frac{4J_r g}{G r_3^2}\right) \qquad (21)$$

It is noted:

$$\delta_{rot}^r = 1 + \frac{4J_r g}{Q r_3^2} \qquad (22)$$

which represents the coefficient of the masses in the rotation motion of the trailer.
The system of forces and moments and the moment which acts on the trailer reduced in the point of coupling between the tractor and the trailer is presented in figure 1.6.

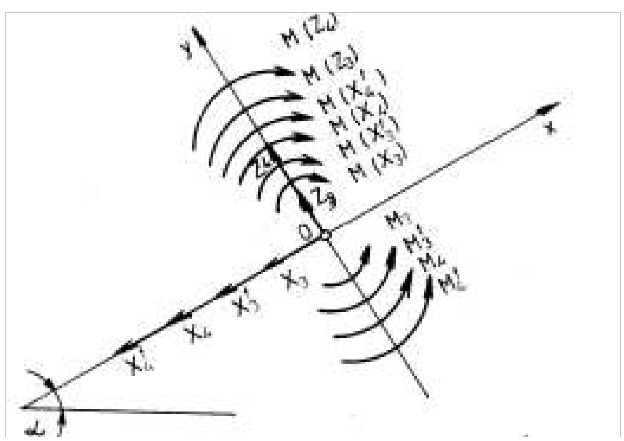


Figure 1.6

The system of forces and moments is reduced in the point 0 to a torsor with elements $\vec{R}_i^r$ and $\vec{M}_i^r$ which have the expressions:

$$\vec{R}_i^r = \left(-F_r^i + F_r^a + X_4 + X_3 + X_4^F + X_3^F + Q\sin\alpha\right)\vec{i} + \left(Z_3 + Z_4 - Q\cos\alpha\right)\vec{j}$$

$$\vec{M}_i^r = \left(M_{\left(F_r^a\right)} + \left(M_4, M_3\right) + M_4^F - M\left(X_3^F, X_4^F\right) + M\left(Q\right) - M\left(Z_3, Z_4\right) - M_{\left(F_r^i\right)} - M\left(X_3, X_4\right) + M_3^F + M_4^F\right)\vec{k} \tag{23}$$

From the analyses of the elements of the torsor $\left(\vec{R}_i^r, \vec{M}_i^r\right)$ it results that these are orthogonal because the system of forces has been considered as acting on the plane xoy.

In reality the tractor trailer aggregate moves on roads with bumps and under the action of random factors which determines that in fact the system of forces should be one system of forces reduced in the point 0 assimilated with the spheric articulations in which the resultant $\vec{R}$ stresses the couple and the moment $\vec{M}_0$ determines the degrees of mobility of the trailer in relation to the tractor.

3. CONCLUSIONS

The dynamic study of the tractor trailer aggregate both on the tractor and the trailer has been performed in different conditions of displacement such as: horizontal road, ascending an descending sloping road, breaking on the rear wheels of the tractor, full breaking of the tractor in accelerated and decelerated motion of the aggregate tractor with biaxial trailer.

The analysis of the systems of forces and moments acting on the tractor and trailer led to the stability of the motion equation of every constituent aggregate respectively the tractor equation (12) and trailer equation (21). Thus, the motion equation of the aggregate tractor with biaxial trailer is defined by the equation:

$$F_i^t = \delta_{rot} M\ddot{x} + \delta_{rot}^r M_r \ddot{x}, \tag{24}$$

in which: M – is the mass of the tractor, and M_r - the mass of the trailer.

The reduction of the systems of forces and moments acting on the tractor and trailer determines in the linking couple between the tractor and the trailer a torsor of the systems consisting of the resultant of the forces $\vec{R}$ and of the resultant of the moments $\vec{M}_0$.

The resultant of the forces $\vec{R}_0$ determines the stresses on the linking couple and the resultant of the moments $\vec{M}_0$ determines the potential movement on the trailer in relation to the tractor. These movements,in accordance with the national legislation and international RNTR-2 should be defined by the pitching rotation, rolling rotation and rotation on a horizontal plane. The linking couple between the tractor and the biaxial trailer which allows the respective movements is in the form of a spheric or quasi spheric articulation which allow the three movements simultaneously or sequentially grouped as two or only one at a time. Of course, these movements occur depending on the aspect of the runway profile of the tractor with biaxial trailer aggegate.

4. REFERENCES

[1] Bodea C. M.: „Theoretical and Experimental Studies on the Couplings between the Tractor and the Semitrailer, doctoral thesis”, 2008.
[2] Cândea I.,e.a: “Static, Kinematic and Dynamic Mechanics " Ed. Didactică şi Pedagogică, Ed. Transilvania din Braşov, 1998,2001,2003.
[3] Ciupercă R.: "Self Directional Running Trains”, Ed Terra Nostra, 2009.
[4] Năstăsoiu S., e.a. "Tractors" , Ed. Didactică şi Pedagogică, Bucureşti, 1983.
[5] A. Nedelcu "Suspension Systems for Agricultural Trailers, Ed Terra Nostra, 2008.
[6] L. Popa, "Breaking Systems for Agricultural Trailers, Ed Terra Nostra, 2008
[7] Rădoi M., Decin E.: "Mechanics", EDP, RA-Bucharest, 1993.
[8] Tecuşan N., Ionescu E.: "Tractors and Cars," Ed "Miron", Timisoara, 1995.