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# CONSIDERATIONS ON MECHANICAL CONTACT WITH IDEAL CONNECTIONS

**Cristina OPRIȚESCU[1], Amalia ȚÎRDEA[2], Mihai Ilie TOADER[3]**
ICECON Bucureşti ~ Research department of Timişoara,
Address: Str. Petre Râmneanţu, nr.2, room 102, Timişoara, ROMANIA, Tel/Fax: 0256 494 944
[1]opritescucristina@yahoo.com, [2]amalia_tirdea@yahoo.com, [3] toader@mec.upt.ro

***Abstract**: Are studied the contact with impact of a mechanical system taking into account the Hertz law. The Hertz contact problem is made assuming the contact force is the sinusoidal. Finally a case study is given. The samples are made of steel with the same physical and mechanical properties.*
***Key words:** impact, Hertz law, vibrations*

## 1. INTRODUCTION

For the theoretical analysis of the mechanical shocks simulation to the flexible models are used especially elastic elements made steel, aluminum, rubber. The metal elastic elements made in steel are presented as coil springs or lamellar and are characterized usually by constant coefficients of rigidity, but in reality these are variables. Energy dissipation in these elements is low, so, to crossing trough resonance of the model, the movement achieves high levels.

Rubber elements or composite materials have different geometric shapes requested usually to compression and shear. During the contact of two elastic bodies, these damping elements used are under the influence of various forces given by Herz law.

## 2. THEORETICAL ANALYSIS OF THE MECHANICAL SHOCKS SIMULATION

The difference of the two bodies movements is represented by

$$\alpha(t) = K\left[f(t)\right]^{\frac{2}{3}} \qquad \textbf{(1)}$$

where $K$ is the Hertz constant given by relation

$$K = \frac{4}{3}\left[\frac{q}{Q_1} + Q_2\sqrt{A+B}\right] \qquad \textbf{(2)}$$

where q, $A$, $B$ are constants that depend on local geometry of the contact region
For example, in the case of a sphere of radius $R_1$ in collision with the surface of a contact plan, the constants are

$A = B = \dfrac{1}{2R_1}$ and $q = \pi^{\frac{1}{3}}$.

$Q_1$ and $Q_2$ depend of Poisson constant $\nu_i$ and of elasticity modules $E_i$ of the two bodies in contact.

$$Q_1 = \frac{1-\nu_1^2}{E_1\pi} \qquad \textbf{(3)}$$

$$Q_2 = \frac{1-\nu_2^2}{E_2\pi} \qquad \textbf{(4)}$$

The impact problem consists in a bar located in a horizontally plane hit by a sphere. The both bodies are considerate made of steel with dimensions like in Figure 1b).

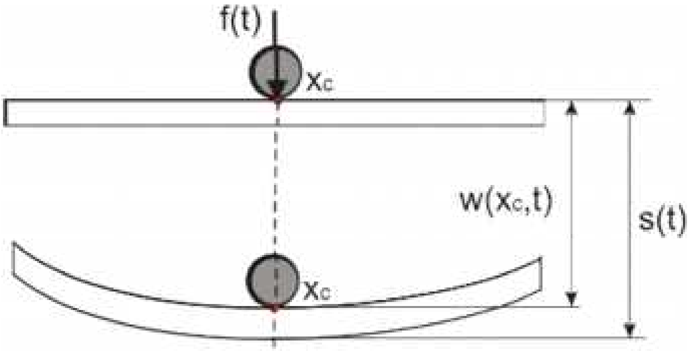


**Figure 1-** a) A bar located in a horizontally plane hit by a sphere

The bar is situated in a horizontally plane is hit by a sphere with mass *m* falling from height *h* with initial speed $v_0$, for which a law (1) is true.

$$\alpha = s(t) - w(x_C, t) \tag{5}$$

The $w(x_C, t)$ (Figure 1a) is the deflection of the bar in the contact point $x_C$ and $s(t)$ is the is the movement of sphere under the action of the contact force $f(t)$. So,

$$s(t) = v_0 t - \frac{1}{m}\int_o^t f(\tau)(t-\tau)d\tau \tag{6}$$

From (1), (5) and (6) is obtained the nonlinear integral equation

$$K[f(t)]^{\frac{2}{3}} = v_0 t - \frac{1}{m}\int_0^t f(\tau)(t-\tau)d\tau - w(x_C, t) \tag{7}$$

Forced vibrations produced in bar by the force variation in the contact point are expressed in terms of normal modes of vibration. For example if a bar is symmetrical supported, the deflection characteristics are $\sqrt{\frac{2}{l}}\sin(\frac{n\pi x}{l})$ for *n=1,2,3,...* where *x* is the is the coordinate point on the bar where occurs the collision. Accordingly, the natural modes of vibration frequencies are $\omega_n = n^2\omega_1$. Central deflection $w(x_C, t)$ is given by

$$w(x_C, t) = \sum_{n=1,3,5,...}^{\infty} \frac{2}{ql}\int_0^t f(\tau)\frac{\sin\omega_n(t-\tau)}{\omega_n}d\tau \tag{8}$$

Where *q* is the mass per unit length and $\frac{ql}{2} = M$ is the reduced mass of the bar.

From (7) and (8) results

$$K[f(t)]^{2/3} = v_0 t - \frac{1}{m}\int_0^t f(\tau)(t-\tau)d\tau - \sum_{n=1,3,5,...}^{\infty} \frac{1}{M}\int_0^t f(\tau)\frac{\sin\omega_n(t-\tau)}{\omega_n}d\tau \tag{9}$$

In practice, the surfaces of the bodies in the collision have a spherical form or more or less close to the sphere, and the impact duration is very short

## 3. SIMULATION EXAMPLE USING ANSYS PROGRAMMING ENVIRONMENT

For the simulation example was used Ansys programming environment and was considered a metallic sphere with mass m in a free fall, from different heights h, on a metallic bar which is in rest in a horizontally plane.

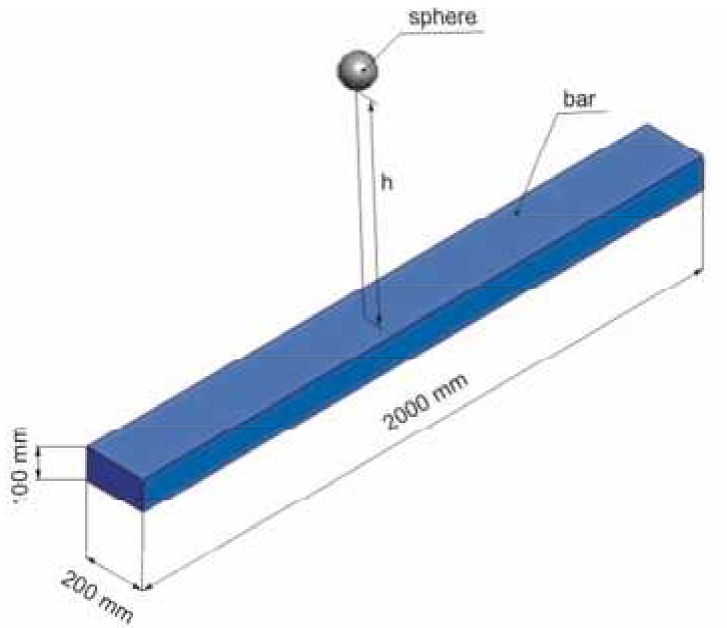


**Figure 2** b) A bar located in a horizontally plane hit by a sphere ~ dimension of bodies in collision

In the Figure 2 and 3 are illustrated the deformations in a bar in rest, hit by a sphere which falls from height *h=50mm*, respectively *h=600mm,* and it can see three situations during the process of collision, for the two situations: a) the initial time with position of the two bodies; the sphere is falling from the *h* height; b) the time when two bodies come into contact; c) the time when the bar has the maximum deflection.

In the graphics from the figure 4 and 5 are represented deflection and stress variation in one node of the bar when the sphere is falling from the h=50 mm, respectively from h=600 mm, obtained by simulation with ANSYS.

Local deformations that occur near the contact area of the bodies in collision influence significantly the process of collision. This meaning that they lead to attenuate the effect of collision, increasing its duration and decrease the maximum contact forces.

If the collision time T is small compared with its oscillation period of the system, it is possible to calculate local deflection.

From the analysis of simulation results that when the sphere falls freely from a height greater, the solicitation along y axis is different like a structure compared when the height is smaller. Also results a amortization of this stress, faster when the height is smaller

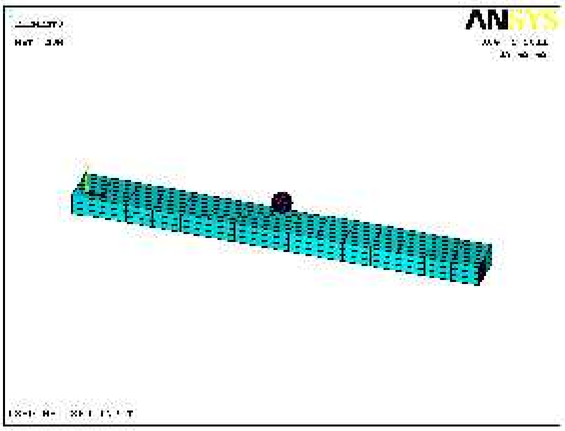


**Figure 2a)** The initial time with position of the two bodies; the sphere is falling from the height *h=50mm*

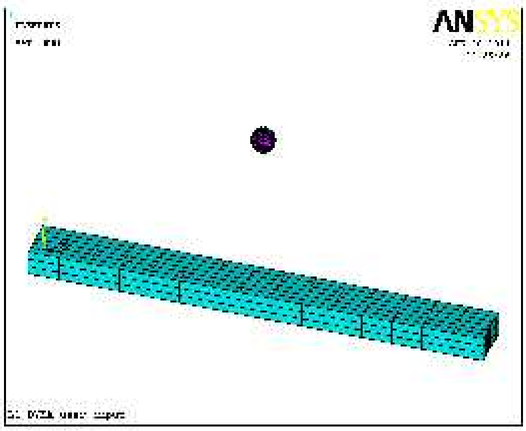


**Figure 3a)** The initial time with position of the two bodies; the sphere is falling from the height *h=600mm*

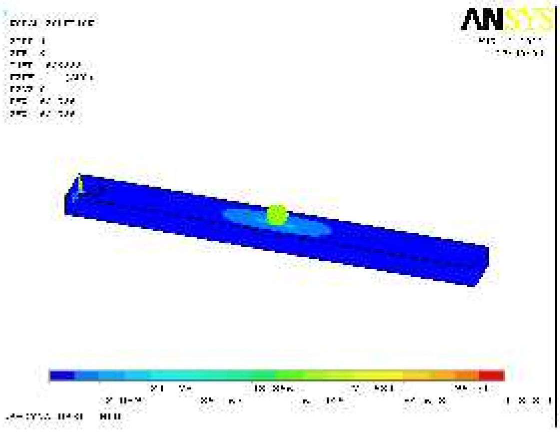


**Figure 2b)** The two bodies come into contact

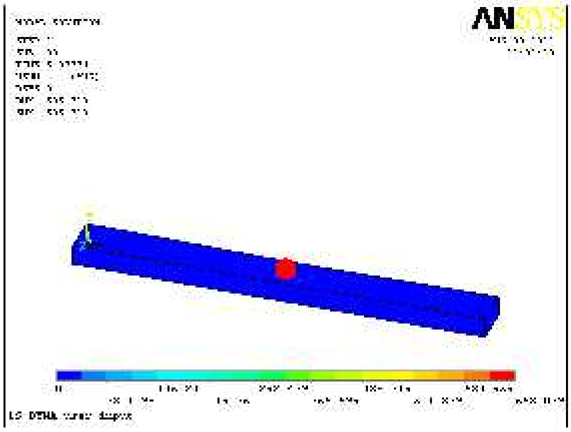


**Figure 3b)** The two bodies come into contact

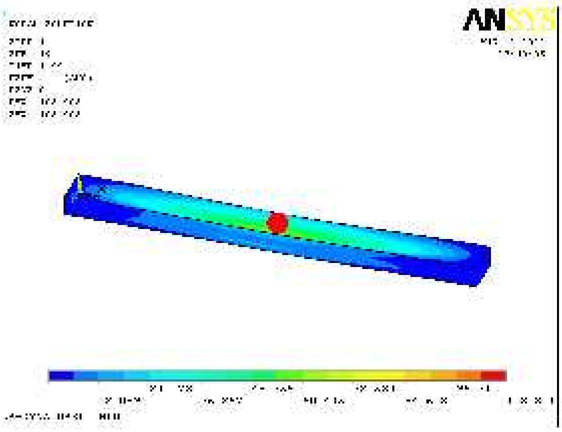


**Figure 2c)** The bar has the maximum deflection

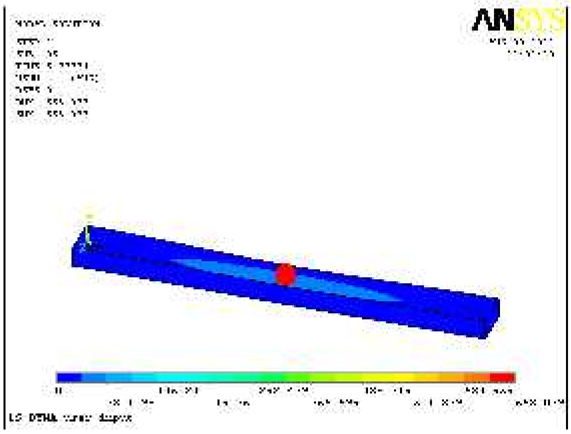


**Figure 3c)** The bar has the maximum deflection

**Figure 2** The deformations in a bar in rest, hit by a sphere which falls from height h=50mm

**Figure 3** The deformations in a bar in rest, hit by a sphere which falls from height h=600mm

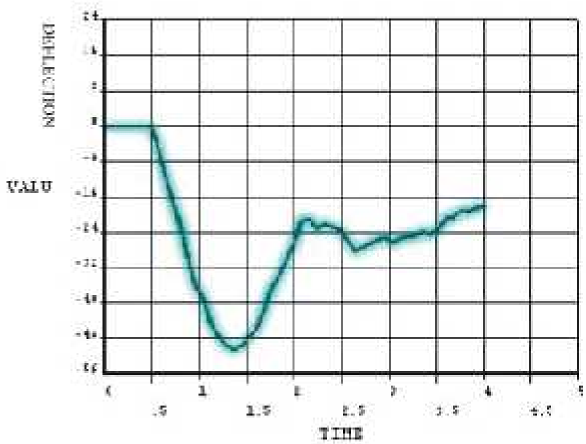


**Figure 4 a)** *Deflection variation* in one node of the bar when the sphere is falling from the *h=50 mm*

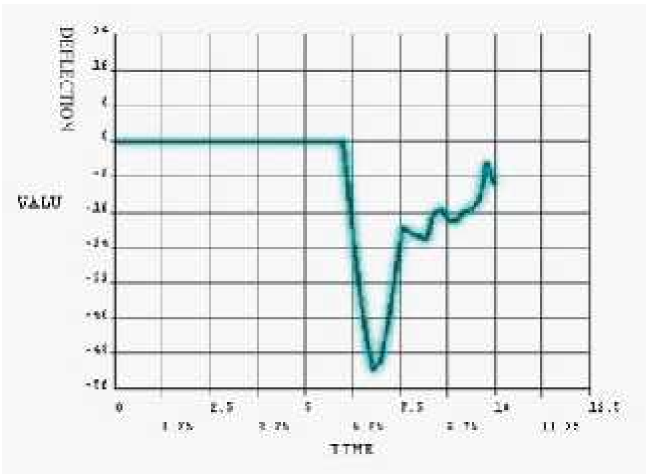


**Figure 5 a)** *Deflection variation* in one node of the bar when the sphere is falling from the *h=50 mm*

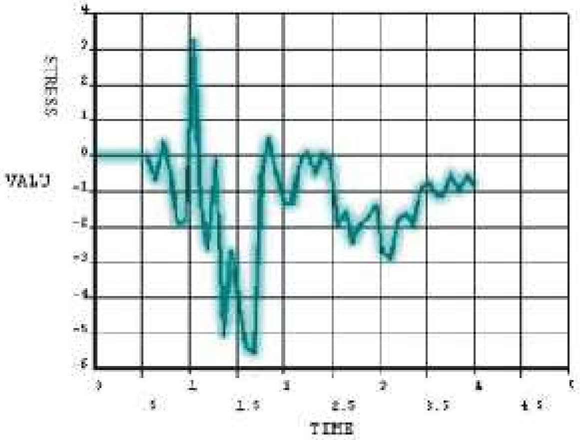


**Figure 4b)** *Stress variation* in one node of the bar when the sphere is falling from the *h=50 mm*

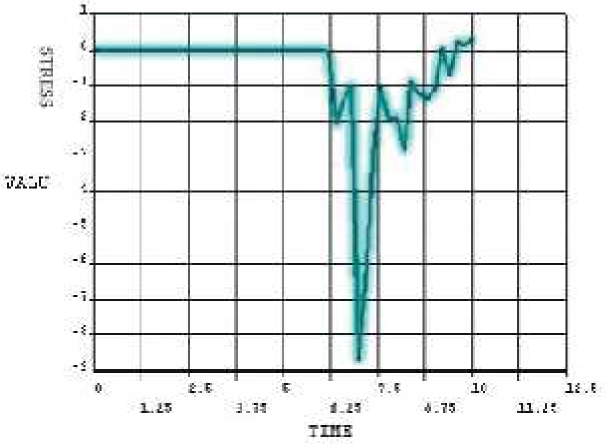


**Figure 5b)** *Stress variation* in one node of the bar when the sphere is falling from the *h=600 mm*

## 4. CONCLUSIONS

The analysis from this paper refers to composite materials but does not avoid the known classical materials. Application of conditions of the Hertz's theory is good for composites and classical materials and this makes as theoretical results can be modeled in a programming language. This language allows underscored the results by the example presented in the paper

The theoretical method is adapted for the case where hertz's contact is well based in the theory of ordinary differential equations, methods known in the study of impact of two bodies. This is reason for which the example of simulation is given by the free fall of a sphere with a mass and from different heights on a metal bar at rest on a horizontal plane

These simulations can be applied to different types of composite materials and, especially to the composite material used for the parapets which need to be mounted in the sides of highways from country

The paper is a continuation of previous research of the authors, which have been well received in various scientific sessions of the country. It is noteworthy that this paper is an extension of scientific ctivities accomplished by CEEX between the Polytechnic University of Timisoara and Transilvania University of Brasov

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