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STOCHASTIC MODEL OF THE NONLINEAR FLUTTER WITH ONE DEGREE OF FREEDOM

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Abstract: *This paper studies a flutter dynamic system type or car in case when F(t) is a random force. Graph of F(t) show the random value of road-wheel contact. The study of damping, aircraft dynamics efficiency and dynamics of car (for suspension) under random perturbation due to wind or current is made by stochastic methods.*
Keywords: *stochastic methods, random noise, flutter dynamic type system.*

1. INTRODUCTION

The theory of the stochastic approximation is a challenging subject of the contemporary science. One the topics of hight interest in this is the nonlinear flutter. The flutter phenomenon is to stabilize and absorb vibrations of a plane landing on a random field. On landing the aicraft thrust acts and due to wind resistance [1], [2], [5].

Flutter is a very important subject; it is in fact a violent oscillation of wings, that usually leads to catastrophic failure.

The same problem can put a jet landing on the water surface or a moving car. It is therefore necessary to study the conditions of stability or instability depending on random parameters.

In this paper we shall present an analysis of a nonlinear stochastic models for dynamic systems in connection with the flutter instability.

2. THE FLUTTER PHENOMENON

We consider a dynamical system which can rotate in the direction of wind, whose velocity is W. This simple type of classical flutter model is in fact a dynamic system with a single degree of freedom. The motion of system is restricted by an elastic spring of constant c and by a linear damping device of constant k. The system, of weight *m*, has a point fixed and makes the angle θ with respect to the wind motion, this angle being in fact the angle of attack [4].

Under these conditions, we consider the following equation of motion of the system:

$$m\ddot{x} = F(t) + Q(x,\dot{x}), \; Q(x,\dot{x}) = -cx - k\dot{x} \qquad (1)$$

where $Q = -cx - k\dot{x}$ forces are:

- cx - elastic oscillator to minimize contact with the ground rigid;
- $k\dot{x}$ - dissipative viscous damper that dissipates (absorbs) oscillations;
- $F(t)$ - random disturbing force representing terrain.

Thrust consists of the result of the relative speed V_r and wind speed W. In the study of the critical speed occurs when wind speed is equal to the relative speed – to balance. Here we ask two questions:

1. the efficiency forces $Q = -cx - k\dot{x}$ contributing to mobile when traveling over rough terrain with a button random . The study is done by stochastic methods and procedures.

2. the stability of dynamic system through mediation and center (statistical operation) was represented by the function $F(t)$ of equation of motion. If we consider $F(t)$ runway, it is replaced by the sine law started with a $F(t) = H\, sin\omega t$.

The equation is written to a fixed reference point with vertical axis *Ox*:

$$m\bar{a} = \bar{F}_e + \bar{F}_a + \bar{F}_p \qquad (2)$$

where:

$\bar{F}_e$ – elastic force;
$\bar{F}_a$ – damping force;
$\bar{F}_p$ – disturbing force;

Further: $|\bar{F}_e| = cx \;\Rightarrow\; \bar{F}_e = -c\bar{r}$;
$|\bar{F}_a| = k\dot{x} \;\Rightarrow\; \bar{F}_e = -k\dot{r}$.

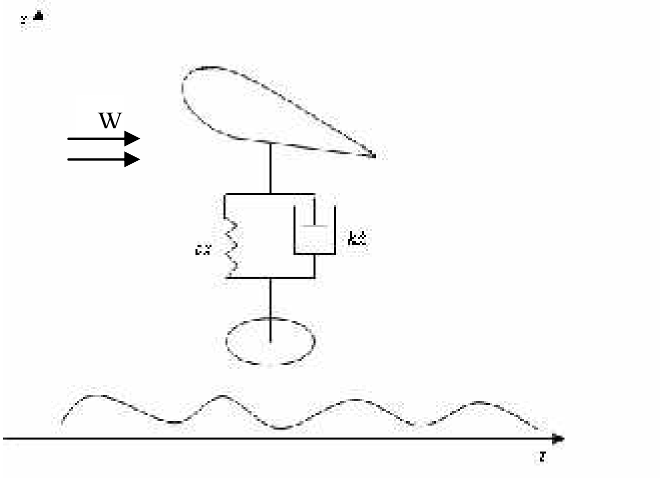

Figure 1: The dinamic system of flutter type

Dividing by *m*, the mass of sistem, equation (1) we get:

$$\ddot{x} + 2\gamma\dot{x} + \lambda^2 x = \frac{H}{m}\sin\omega t \qquad (3)$$

We noted the damping force $2\gamma\dot{x}$, with $\lambda^2 x$ elastic force and $\frac{1}{m}F(t)$ random disturbance force multiplying. We believe that disruptive force, in probabilistic terms must be of the form:

$$F(t) = H\, sin\omega t \qquad (4)$$

Experimental data of spectral density $F(t)$ is know as:

$$S_F(\omega) = \begin{cases} S_0 & 0 \le \omega \le \omega_0 \\ 0 & \omega > \omega_0 \end{cases} \qquad (5)$$

Accordingly we have random dispersion interaction:

$$\sigma_F^2 = \int_0^\infty S_F(\omega)d\omega = \begin{cases} S_0\omega & 0 \le \omega \le \omega_0 \\ 0 & \omega > \omega_0 \end{cases} \qquad (6)$$

In this case, frequency ω_0 is called frequency cut. Looking from statistically random function $F(t)$, we obtain that is a stationary random process with zero mean and variance given by (6), [6].
In solving equation (3) we seek the general solution as the sum of two individual root [7]:

$$x = x_g^o + x_p^n \qquad (7)$$

To do this, solve the equation feature:

$$\ddot{x} + 2\gamma\dot{x} + \lambda^2 x = 0 \quad\Rightarrow\quad r^2 + 2\gamma r + \lambda^2 = 0 \qquad (8)$$

hence the roots:

$$r_{1,2} = -\gamma \pm \sqrt{\gamma^2 - \lambda^2} \qquad (9)$$

but $\gamma < \lambda$ because $F_a < F_e$ dissipative force is less than the elastic. Therefore, $r_{1,2} = -\gamma \pm i\delta$, noted by $\delta^2 = \gamma^2 - \lambda^2$.
These roots lead us to two particular solutions of equation (8). In these conditions we get:

$$x_g^o = e^{-\gamma t}\{c_1\cos\delta t + c_2\cos\delta t\} \qquad (10)$$

The term x_g^o dissipates (stabilizes) quickly because $\lim_{t\to\infty} e^{-t} = 0$ and most important will be the solution that influence particular disturbance.
If $x_p^n = M\sin\omega t + N\cos\omega t$ determine M, N introducing in homogeneous equation by identifying

$x_p^n = M\left\{\sin\omega t + \frac{N}{M}\cos\omega t\right\} = \frac{M}{\cos\theta}[\sin\omega t \cdot \cos\theta - \cos\omega t \cdot \sin\theta]$, $\frac{N}{M} = tg\varphi = -tg\theta$

After calculations we find:

$$x_p^n = \frac{N}{m\sqrt{(\lambda^2-\omega^2)^2+4\gamma^2\omega^2}}\sin(\omega t - \theta) \quad (11)$$

Returning to the solution of the equation $x = x_g^o + x_p^n$, x_g^o is canceled and $x = x_p^n$.

Comments:

- x_p^n can and Laplace transform with zero conditions;
- for γ small ($\gamma \to 0$), if $\lambda^2 \to \omega^2$ we have resonance phenomenon that increases the amplitude $\ddot{x} + \omega^2 x = \frac{N}{m}\sin\omega t$. In this case $x_p^n = Mt\sin\omega t$ and amplitude increase as $Mt \to \infty$ because the elastic shock *cx* resonate with the way the euation $\varepsilon\sin\omega t$ [3].

3. STOCHASTIC MODEL OF THE NONLINEAR FLUTTER

As show in Fig. 1, the structural dynamics of the system is described by a single equation of force equilibrium:

$$m\ddot{x} + c\dot{x} + kx = \frac{1}{2}\rho W^2 SC(\theta) \quad (12)$$

As the lift force is $L = \frac{1}{2}\rho W^2 SC(\theta)$; ρ is the density of fluid, S is the surface of the plate exposed to the fluid flow and $C(\theta)$ is the lift coefficient. (*fig.1).*

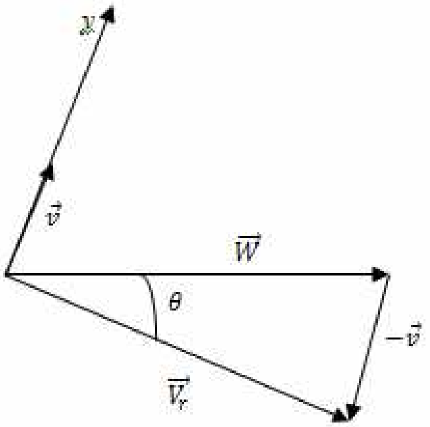

Figure 2 Dynamic system structure

According to *fig. 2* the relative velocity has the magnitude $V_r = \sqrt{\dot{x}^2 + W^2}$ and makes angle with the axis *OX* given by $tg\theta = \frac{\dot{x}}{W}$. The aerodynamic coefficient $C(\theta) = C(\dot{x})$ may be described with a good approximation, by the following polynomial representation

$$C(\theta) = a_1\theta - a_2\theta^3 + a_3\theta^5 - a_4\theta^7 + \dots \quad (13)$$

Indicated by analytical and experimental. For small angles of attack approximating $tg\theta \approx \sin\theta \approx \theta$ we may use for low frequencies a quasisteady reprezentation of the lift coeficient. For example,

$$\frac{1}{2}\rho W^2 SC(\theta) = \frac{1}{2}\rho W^2 S\left[a_1\frac{\dot{x}}{W} - a_3\left(\frac{\dot{x}}{W}\right)^3 + a_5\left(\frac{\dot{x}}{W}\right)^5 - a_7\left(\frac{\dot{x}}{W}\right)^7 + \cdots\right] \quad (14)$$

Passing into the phase space *(x,y)* where $y = \dot{x}$ and using (1) and (3), we obtain the following system of 1st order ODE:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -\frac{kx}{m} + \frac{1}{m}\left(\frac{1}{2}\rho WSa_1 - C\right)y - \frac{1}{2}\rho\frac{Sa_3}{mW}y^3 + \frac{1}{2}\rho\frac{Sa_5}{mW}y^5 \dots \end{cases} \quad (15)$$

In the case of equilibruim in the first approximation, we have the solution $y = 0$. Using the expression of the critical velocity $V_c = \frac{2C}{\rho Sa_1}$ and introducing the notations: $a = \frac{W}{V_c}$, $b = \frac{K}{m}$ the system (15) becomes:

$$\begin{cases} \dot{x} = y = F_1(x,y) \\ \dot{y} = -bx + y\{-1 + a[1 - f(y)]\} = F_2(x,y) \end{cases} \quad (16)$$

Where

$$f(y) = \frac{A_3}{A_1}y^2 - \frac{A_5}{A_1}y^4 + \frac{A_7}{A_1}y^6 \text{ şi } V_c = \frac{2C}{\rho SA_1} \quad (17)$$

The equilibrium critical point of the non-linear system, for $F_1 = 0$ şi $F_2 = 0$ becomes $O^*(x^* = 0, y^* = 0)$. We will deal with the stability around this point and, due to the complexity of the non-linear system, we will use various criteria such as the stability in firt approximation.

4. STABILITY IN FIRST APROXIMATION

The Jacobian of the system (16) is:

$$J = \begin{pmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -b & -1 + a[1 - f(y) - yf'(y)] \end{pmatrix} \quad (18)$$

From (16) we obtain the linear system:

$$\begin{cases} \dot{x} = y \\ \dot{y} = -bx + y(-1 + a) \end{cases} \quad (19)$$

where:

$$J(P^*) = \begin{pmatrix} 0 & 1 \\ -b & a - 1 \end{pmatrix} \quad (20)$$

Denoting by $P(r)$ the characteristic polynomial, the equation $P(r) = 0$ becomes:

$$P(r) = det(J - rI) = r^2 + (1 - a)r + b = 0 \quad (21)$$

The study reduces to compare the wind speed W with V_C , i.e.:

- if $0 < a < 1 \Leftrightarrow W < V_C$, where $a = \frac{W}{V}$ the linear system (19) is asymptotic stable about the point O^* and the non-linear system (16) is asymptotic stabil.
- if $a > 1$ the linear system (19) is not stable, implying thus that for $W > W_{cr}$ the system (16) is not stabil..
- if $a = 1,\ W_{cr} = V_c$ we have simple stability for the system (19) and one cannot decide on the stability of the non-linear system (16).

Analyzing the sign of the expression $\Delta = (1 - a)^2 - 4b$ in the plane *(a, b)* we can characterize the stability of the traiectories:

1. if $\Delta > 0,\ D_1 = \left\{(a, b) \middle| 0 < a < 1,\ b < \frac{(1-a)^2}{4}\right\}$ whith the initial condition $t = 0$, $P_0(x_0, y_0)$ the traiectories of the system (16), $x = x(t), y = y(t)$ are curves which tend asymptotically to the stable point O^*.
2. if $\Delta < 0,\ D_2 = \left\{(a, b) \middle| 0 < a < 1,\ b > \frac{(1-a)^2}{4}\right\}$ then the traiectories of the system (16), $x = x(t, x_0, y_0),\ y = y(t, x_0, y_0)$ are spirals tending asymptotic to the point O^*.
3. for $a > 1$ we have instability.
4. the critical case $W = V_{cr},\ a = 1, \Delta = 0$ (on the parabola P) the reduced linear system $\dot{x} = y, \dot{y} = -bx$ has trajectories tending to an ellipse (E):

$$E: \begin{cases} x = x_0 \cos\sqrt{bt} + \frac{y_0}{\sqrt{b}} \cos\sqrt{bt}, \\ y = -x_0 \sin\sqrt{bt} + y_0 \cos\sqrt{bt} \end{cases} \quad (22)$$

Which are simple stable concerning the center O^*; nothing can be said about the stability of the system (16).

5. CONCLUSION

To this end, we mention that our previous analysis may be extended by taking into account the damping of the system, which will influence its state of stability/instability, but this task is left for a future study.

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