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NUMERICAL COMPUTATION OF THE DYNAMIC CHARACTERISTICS OF A MULTILEVEL STRUCTURE

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Abstract: *Due to the essential need of investigating the dynamic characteristics of multilevel structures this work was focused on studying this behavior. The designed structure consists of three levels. The ground flour is higher than the other two levels. A Matlab software was used in the dynamic calculations of the proposed design. The free vibrations behaviour of the discrete elastic system was deduced depending on acceleration vector, damping matrix, velocity vector, and stiffness matrix. A prominent finding was that the proper location of dampers is at 0.533 m of the scaled height. It was found that it is easier to detect the deflections at any point in the structure. Finally the main benefit was that the damping coefficient increased by 4.2% and the dissipated energy (caused by earthquake) increased by 19%.*

1. FREE VIBRATIONS OF DAMPED STRUCTURES

The free vibrations of a discrete system with *n* dynamic degrees of freedom (dof) can be described by the equation:

$$\mathbf{M}\ddot{\mathbf{a}} + \mathbf{C}\dot{\mathbf{a}} + \mathbf{K}\mathbf{a} = \mathbf{0}\,. \qquad \textbf{(1)}$$

Where:
M is the matrix of the mass of the system;
$\ddot{\mathbf{a}}$ is the vector of the accelerations;
C is the damping matrix;
$\dot{\mathbf{a}}$ is the vector of velocities;
K is the stiffness matrix of the structure.
The solutions of the equation (1) are of the form:

$$\mathbf{a} = \mathbf{v}e^{\omega_a t}\,, \qquad \textbf{(2)}$$

where vectors **v** of the forms of vibratiom, natural circular frequencies of the damped system ω_a are complex cantities. Substituting (2) in equation (1) a generalized eigenproblem results:

$$(\mathbf{K} + \omega_a \mathbf{C} + \omega_a^2 \mathbf{M})\mathbf{v} = \mathbf{0}. \qquad \textbf{(3)}$$

The dimension of the matrices from (1) is *nxn* and vectors have *n* components.
Matrices **M**, **C** and **K** for the whole structure result from an assembling process given by the relations:

$$\mathbf{M}_{\mathbf{ind,ind}}=\mathbf{M}_{\mathbf{ind,ind}}+\mathbf{m}$$
$$\mathbf{C}_{\mathbf{ind,ind}}=\mathbf{C}_{\mathbf{ind,ind}}+\mathbf{c} \qquad \textbf{(4)}$$
$$\mathbf{K}_{\mathbf{ind,ind}}=\mathbf{K}_{\mathbf{ind,ind}}+\mathbf{k}\,.$$

Where vector **ind** contains the indices of the displacements of the current beam element.
Ecuations (4) are repeated for each element.
Concentrated masses are introduced by the relation:

$$\mathbf{M}_{\mathbf{inm,inm}}=\mathbf{M}_{\mathbf{inm,inm}}+m. \qquad \textbf{(5a)}$$

Where vector **inm** conains the indices of translational displacements of the current node and *m* is the value of corresponding mass. Relation (5a) is repeated for each node where concentrated mass is applied.
Local damping effects are introduced by:

$$\mathbf{C}_{jnd,jnd}=\mathbf{C}_{jnd,jnd}+c\,. \qquad \textbf{(5b)}$$

Where *jnd* is the index of the displacement corresponding to the external damper and c is its damping factor. Relation (5b) is repeated for each dof with external damping.
The supports of the structure are introduced by the relation:

$$\mathbf{K}_{jnd,jnd}=\mathbf{K}_{jnd,jnd}+r\,. \qquad \textbf{(5c)}$$

Where *jnd* is the index of the constrained displacement of the support and *r* is its the stiffness. Equation (5c) is repeated for each simple support restriction.
The matrices of the beam elements are:

$$\mathbf{m} = \mathbf{R}^T \overline{\mathbf{m}} \mathbf{R}$$
$$\mathbf{c} = \mathbf{R}^T \overline{\mathbf{c}} \mathbf{R} \quad \textbf{(6)}$$
$$\mathbf{k} = \mathbf{R}^T \overline{\mathbf{k}} \mathbf{R}$$

Here, matrices $\overline{\mathbf{m}}$, $\overline{\mathbf{c}}$, $\overline{\mathbf{k}}$ are expressed in the local reference system of each element and **R** is the rotation matrix.

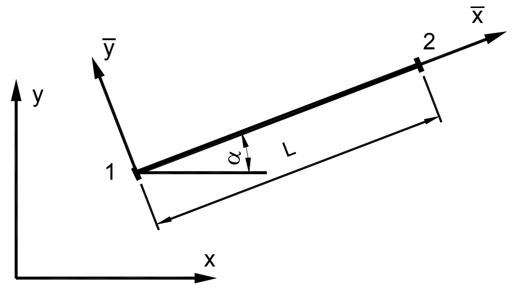

Figure 1: The beam element

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_0 \end{bmatrix}, \quad \textbf{(7)}$$

with

$$\mathbf{R}_0 = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The matrices of the beam in the local reference system result using the displacement fields of the beam. For simplicity we assume that the local system of the beam coincides with the structural reference system (fig. 2.).

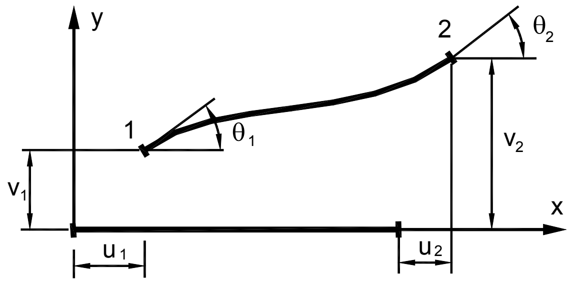

Figure 2: Displacements of the beam element

The displacement field of the beam can be expressed in polynomial form:

$$u = \alpha_1 + \alpha_2 x,$$
$$v = \alpha_3 + \alpha_4 x + \alpha_5 x^2 + \alpha_6 x^3. \quad \textbf{(8a)}$$

or

$$\mathbf{u} = \begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} 1 & x & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & x & x^2 & x^3 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_6 \end{Bmatrix} = \mathbf{P}\boldsymbol{\alpha}. \quad \textbf{(8b)}$$

Parameters $\boldsymbol{\alpha}$ can be determined expressing the displacements of the nodes of the beam:

$$\mathbf{a} = \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & L & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & L^2 & L^3 \\ 0 & 0 & 0 & 1 & 2L & 3L^2 \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{Bmatrix} = \mathbf{C\alpha}. \tag{9}$$

Expressing the internal displacements of the beam from (8) and (9) results:

$$\mathbf{u} = \mathbf{PC}^{-1}\mathbf{a} = \mathbf{Na}\,. \tag{10}$$

Where matrix **N** contains the shape functions of the beam displacements.
The strains can be expessed as:

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon \\ \chi \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial}{\partial x} & 0 \\ 0 & \dfrac{\partial^2}{\partial x^2} \end{bmatrix} \mathbf{Na} = \mathbf{Ba}\,. \tag{11}$$

The stresses are:

$$\boldsymbol{\sigma} = \begin{bmatrix} N \\ M \end{bmatrix} = \mathbf{DBa}\,. \tag{12}$$

Here

$$\mathbf{D} = \begin{bmatrix} EA & 0 \\ 0 & EI \end{bmatrix}. \tag{13}$$

In equation (13) E is the elasticity modulus of the material of the beam, A is the transversal section area and I is its moment of inertia.
The mass, damping and stiffness matrices of the beam can be obtained by the relations:

$$\mathbf{m} = \rho A \int_0^L \mathbf{N}^T \mathbf{N} dx = \rho A \mathbf{m}_0,$$
$$\mathbf{c} = \psi A \int_0^L \mathbf{N}^T \mathbf{N} dx = \psi A \mathbf{m}_0, \tag{14}$$
$$\mathbf{k} = \int_0^L \mathbf{B}^T \mathbf{DB} dx.$$

In relation (14) ρ is the density of the material and ψ is the damping factor.
Damping devices with a damping factor am_d can be treated as special beam elements with the following damping matrix expressed in the local system of reference:

$$\mathbf{c} = \begin{bmatrix} am_d & 0 & 0 & -am_d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -am_d & 0 & 0 & am_d & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{15}$$

Using a symbolical computation, relations (8)...(14) give the consistent mass and stifness matrix of the beam:

```
m0=
   [    1/3*L,            0,             0,    1/6*L,            0,            0]
   [        0,      13/35*L,  11/210*L^2,        0,       9/70*L, -13/420*L^2]
   [        0,   11/210*L^2,   1/105*L^3,        0,   13/420*L^2,   -1/140*L^3]
   [    1/6*L,            0,             0,    1/3*L,            0,            0]
   [        0,       9/70*L,  13/420*L^2,        0,      13/35*L, -11/210*L^2]
   [        0, -13/420*L^2,   -1/140*L^3,        0, -11/210*L^2,    1/105*L^3]

k=
   [  1/L*EA,            0,            0, -1/L*EA,            0,           0]
   [        0,  12/L^3*EI,    6/L^2*EI,        0, -12/L^3*EI,    6/L^2*EI]
   [        0,    6/L^2*EI,      4/L*EI,        0,   -6/L^2*EI,      2/L*EI]
   [ -1/L*EA,            0,            0,  1/L*EA,            0,           0]
   [        0, -12/L^3*EI,   -6/L^2*EI,        0,  12/L^3*EI,   -6/L^2*EI]
   [        0,    6/L^2*EI,      2/L*EI,        0,   -6/L^2*EI,      4/L*EI]
```

2. PROGRAM *CDCP* – DYNAMIC ANALYSIS OF PLANE FAMES

Program `CDCP` uses the matrix displacement method. The algorithm is described by relations (4) – (7) and (15). The program solves the generalized eigenproblem (3) and gives the results for the first ***nmod*** vibration modes.

3. INTRODUCING THE DATA OF THE STRUCTURE

The following set of data describes the structure presented in figure 3, with two dampers mounted at the first floor. The data for the structure without dampers can be obtained by introducing zero value for the damping. Homogenous units in N and m were used. The gometry of the structure is described by the coordinates of the nodes.

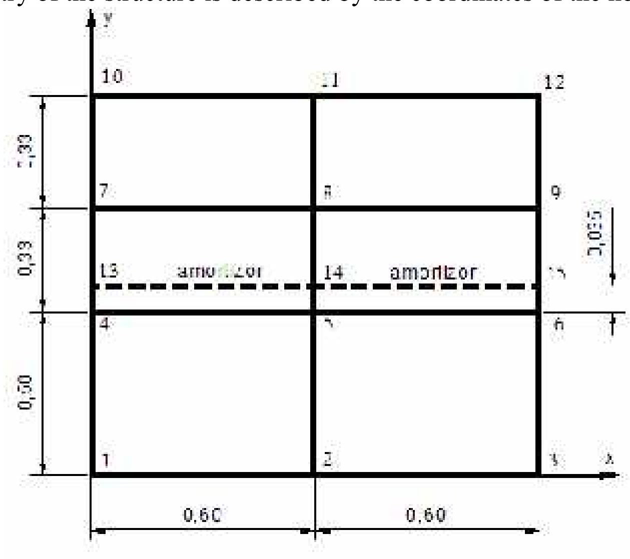

Figure 3: Metal frame
OL 37, E=2,1*10^{11} N/m^2, beams 0,6*0,005 m, columns 0,6*0,003 m

E - is the elasticity modulus in N/m^2 (OL37).
ro – is the density of material in kg/m^3 (ρ).
The value *ro* =206/(4*1.2*.6*.005+3*1.16*.6*.003) = 9969 kg/m^3 introduces the weight of the real model.
ams - is the structural damping factor ψ from equation (14) - here its value is taken zero.
amd – is the damping factor of the dampers - here this value is zero.
A1, I1, A2, I2 - are the characteristics of the section of the beams and columns respectively.
Matrix *El* contains the data for the beam elements in the form:
El=[*nod1 nod2 A I am*] – if the value of the damping factor *am* is nonzero, the beam is considered damper with its damping matrix given by equation (15).
Matrix *cr* – contains the data for the supports in the form: *cr=[node direction stiffness]*.
Matrix *Mn* - contains the concentrated masses in the form: *Mn=[node mass]*.
Matrix *An* - contains concentrated dampings in the form: *An=[node direction damping_factor]*.
nmod – is the number of vibration modes for which results are requested.

4. RESULTS

a. Structure without damping

Table 1: Results for the structure without damping.

Vibration mode	Computed circular frequency (rad/s)	Measured frequency (Hz)	Computed frequency (Hz)	Computed period (s)
1	18,4382	2,78	2,93	0,341
2	69,8089	13,37	11,11	0,090
3	128,0013	24,21	20,37	0,049

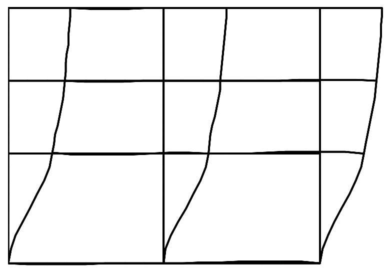

Figure 4: Mode 1; p = 18,44 rad/s (f = 2,93 Hz, T = 0.341 s)

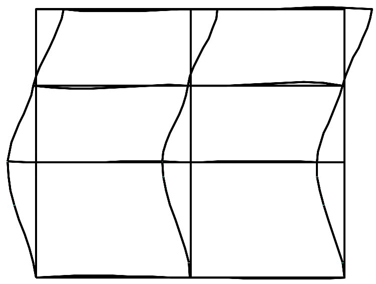

Figure 5: Mode 2; p = 69,81 rad/s (f = 11,11 Hz, T = 0.090 s)

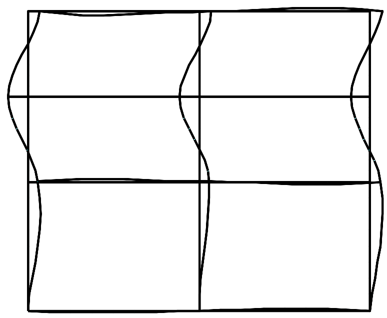

Figure 6: Mode 3; p = 128,00 rad/s (f = 20,37 Hz, T = 0.049 s)

b. The influence of the structural damping

In this case for the experimental value of the damping factor ξ=0,017 in the case of the circular frequency ω_n=17,358 rad/s, the value of damping coefficient is:

$ams = \psi = 2*\xi*\omega_n*\rho$ = 2*0,017*17,358*9969 = 5883 kg/(m^{3}s) = 5883 Ns/m^4.

The results are complex cuantities given in table 2:

Table 1: Results for the structure without damping.

Mode	Without damping (ω_n) [rad/s]	Damped vibrations ($n+\omega_a i$) [rad/s]
1	18.4382	-0.295 – 18.4359i
		-0.295 + 18.4359i
2	69.8089	-0.295 – 69.8082i
		-0.295 + 69.8082i
3	128.0012	-0.295 –128.0009i
		-0.295 +128.0009i

The relationship between circular frequencies is: $\omega_n = \sqrt{n^2 + \omega_a^2}$.

For example, for mode 1 we get: $\omega_{n1} = \sqrt{n_1^2 + \omega_{a1}^2} = \sqrt{0{,}295^2 + 18{,}4359^2} = 18{,}4382$.

The displacement of a point of the structure can be described by the relation.

$u(t) = Ae^{nt}\sin(\omega_a t)$,

where A is the amplitude of the displacement.

In figure 4 is represented the displacement of a point in the fundamental mode for the case A=1 mm.

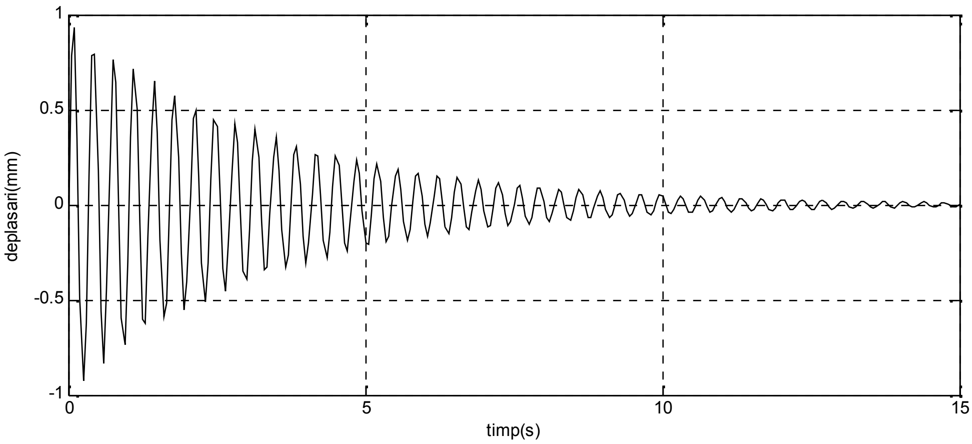

Figure 7: Damped vibrations in the fundamental mode

c. The influence of the dampers

From the experimental measurements resulted that the best placement of the dampers is at the first floor. For this case, in the computational model was introduced the damping coefficient corresponding to the damping factor $\tau = \frac{1}{2\pi}\ln\frac{5}{3.5} = 0.0568$.

The results had significant modifications only at the fundamental mode. The following values resulted:

Table 3: Results for the structure with dampers.

Mode	With structural damping $p+\omega_a$i [rad/s]	With structural damping + dampers $p+\omega_a$i [rad/s]
1	-0.295 - 18.4359i	-0.369 - 19.2104i
	-0.295 + 18.4359i	-0.369 - 19.2104i

5. CONCLUSION

By mounting the dampers, the fundamental frequency increased by 4,2%. Another important fact is that the equivalent damping factor inreases too. For the structure without damping $\xi=p/\omega_a$= 0,295/18,4359=0,016. By mounting the dampers results: $\xi_d=p/\omega_a$= 0,369/19,2104=0,019, which means an increase of 19%. The effect of the increasing of the damping is favorable by the more efficient dissipation of the earthquake induced energy.

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