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THE RUBBER COVERED ROLLERS WHICH ARE WIDWLZ USED IN PROCESS ING MACHINERY MATHEMATICAL MODELS

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Abstract: *The contact of solids have surface layer elastic properties differ from the substrate frequently with occurs in practice for example, the rubber covered rollers which are widely used in processing machinery. The mathematical models is presented in this paper.*
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1.INTRODUCTION

The elastic deformation in the contact region is obtained in the Hertz by assuming each solid deforms as en elastic , isotropic, homogenous half-space. If the material of either solid is anisotropic, or inhomogeneous, or if their thickness are not large compared w Practical examples of contact anisotropic solids are found with single crystals and extended polymers filaments , between inhomogeneous materials with foundations built on stratified rock or soil.

2. LAYERED SOLIDS, PLATES, AND SHELLS

The basic situation is illustrated in (fig. 1a) in which body (2) is in contact with the surface layer (1) on substrate (3) . If the thickness b of the layer is large compared with the contact size 2a, then the substrate has little influence and the contact stresses between (1) and (2) are given by the Hertz theory . In this section we are concerned with the situation in which h is comparable with on less than 2a. There are various possibilities (a), the layer may be bonded maintain contact with the substrate at all points , has be free to slip without frictional restraint (b), at the other extreme the layer may be bonded to the substrate ; (c) slip may occur when the shear traction at interface exceeds limiting friction: and (d) the layer. Initially in complete contact with the substrate, may practically lift from the substrate under load.. The non-conforming contact between the layer and body (2) may also influenced by frictional tractions . Even if the classic constants are the same ($E_1=E_2$, $\nu_1=\nu_2$).limited thickness of the layer results in a relative tangential displacement in the interface which will be resisted . which will be resisted by friction.

If the contact width is small compared width the radii of curvature of the bodies , the curvature of the layer can be ignored in analyzing in deformation and the solid (2) and (3) can be taken to be elastic half-space.

In the case where the layers is everywhere at the layers in contact with a rigid friction substrate interface are $\tau_{xz}=0$ and $u_z=0$. The stress in the layer are then the same ass in on half of a layer of thickness faces (fig.1b) .The stresses in the layer are best expressed in terms Fourier Integral, for which the reader is referred in the books by Sneddon...In this case with an even distribution of pressure applied symmetrically each surface z=+-b, Sneddon shows the normal displacement of each surface is given by

$$\overline{u}_z = \frac{4(1-\nu_1^2)}{\pi E_1}\int_0^\infty \left(\frac{2\sinh^2 ab}{2ab\sinh 2ab}\right) p(\alpha)\frac{\cos\alpha x}{\alpha}\, d\alpha \qquad (1)$$

where $p(\alpha)$ is the Fourier Cosine Transformof the pressure p(x)

$$p(x) =' \int_0^{\infty} p(x)\cos \alpha x \qquad (2)$$

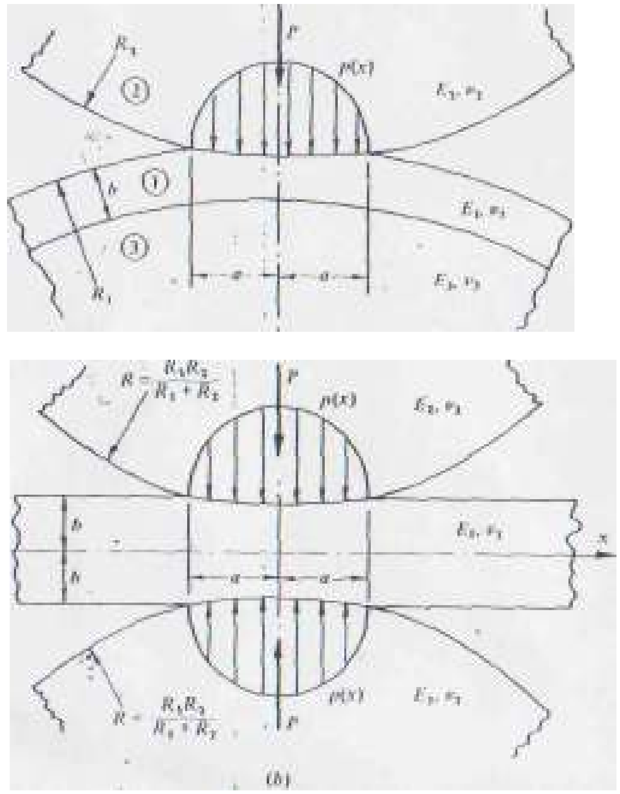

Fig.1

For a uniform pressure p distributed over the interval $-c\angle x\angle c$, equation (1) gives

$$p(\alpha) = (p/\alpha)\sin(\alpha c) \qquad (3)$$

For a triangular distribution of pressure of peak value p_0

$$p(\alpha) = \frac{2p_0}{c\alpha^2}\sin^2(\frac{c\alpha}{2}) \qquad (4)$$

In the limit , as $c\rightarrow 0$, the transform of a concentrated force P is P/2. Frictional traction q(x) on the faces of the layer can be handled in this same way. A thin layer indented by a frictionless rigid cylinder is shown in (fig.2).
If $b\angle\angle a$ is reasonable in the first instance to assume the deformation through the layer is homogeneous , plane sections remain plane after compression show in (fig 2a) , so that the stress σ_x is uniform through the thickness.

We will consider first the case of no friction at the interface between the layer and the rigid substrate whereupon $\sigma_{xs}=0$ throughout.
In plane stress

$$\varepsilon_z = \frac{1-\nu^2}{E}\sigma_z = -\frac{1-\upsilon^2}{E}p(x) \qquad (5)$$

The compressive strain in the element is given by t5he geometry of deformation

$$\varepsilon_z = -(\delta - \frac{x^2}{2R})/b \qquad (6)$$

Since the pressure must fall to zero at $x=\pm a$, equations (5) and (6) give $\delta=a^2/2R$ and

$$p(x) =' \frac{E}{1-\upsilon^2}\frac{a^2}{2Rb}(1 - x^2 / a^2) \tag{7}$$

whence the load

$$p = \frac{2}{3}\frac{E}{1-\upsilon^2}\frac{a^3}{Rb} \tag{8}$$

In the case where the layer is bounded to the substrate plane sections remain plane, the strain ε_x is zero through,

$$\varepsilon_x = \frac{1-\upsilon^2}{E}\left\{\sigma_x + \frac{\upsilon}{1-\upsilon}p(x)\right\} = 0 \tag{9}$$

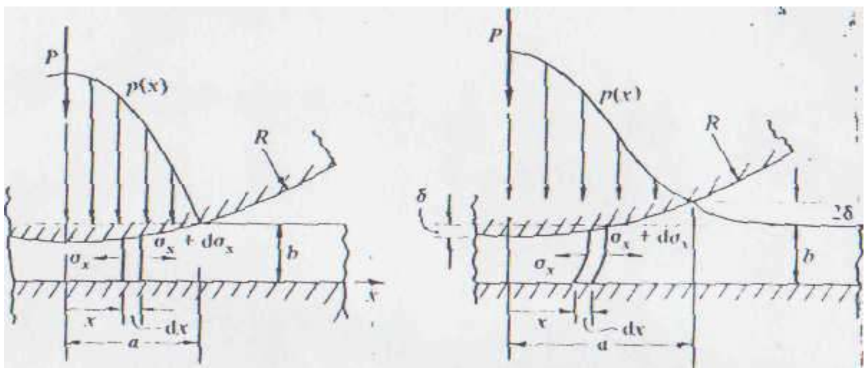

Fig. 2. An elastic layer on a rigid substrate indented liy a rigid cylinder: *{a)* Poisson n's ratio *P<C0A5;U>)* $v = 0.5$.

In this case

$$\varepsilon_x = \frac{1-\upsilon^2}{E}\left\{-\upsilon(x) - \frac{\upsilon}{1-\upsilon}\sigma(x)\right\} \tag{10}$$

If substituting σ_x and ε_x in (6) given

$$p(x) = \frac{(1-\upsilon)^2}{2\upsilon}\frac{E}{1-\upsilon^2}\frac{a^2}{2Rb}(1 - \frac{x^2}{a^2}) \tag{11}$$

And

$$P = \frac{1}{3}\frac{(1-\upsilon)^2}{1-2\upsilon}\frac{E}{1-\upsilon^2}\frac{a^3}{Rb} \tag{12}$$

For a thin layer of an incompressible material we have

$$p(x) = \frac{Ea^4}{24Rb^3}(1 - \frac{x^2}{a^2}) \tag{13}$$

$$p = \frac{2Ea^5}{45Rb^3} \tag{14}$$

$$\delta = a^2 / 6R \tag{15}$$

3. CONCLUSION

Most analyses at the pres time, however, assume the contact to be frictionless and are restricted to either the plane-strain conditions of line contact, or the axi-symmetric contact of solids of revolution in which the contact area is circular.

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